Global relativity establishes absolute time and a universal frame of reference

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It has been known for several decades that the rest energy of all matter in space is essentially equal to the total gravitational energy in space. The Dynamic Universe model introduced in this paper studies the equality as a dynamic zero-energy balance of motion and gravitation in spherically closed space. In such a solution time is absolute, the fourth dimension has metric nature, and relativity appears as the measure of the locally available share of total energy in space. The study of the zero-energy balance in spherically closed space can be based on few postulates and the derivation of predictions for local physical phenomena and for cosmological observations can be carried out with fairly simple mathematics essentially free of additional parameters.

In all interactions in space the total energy is conserved. A clock in motion in space does not lose time because of slower flow of time but because motion in space diverts a share of the total energy of the device thus leaving less energy for the oscillation running the clock. In the DU framework a local state of rest can be related to the state of rest in hypothetical homogeneous space, which serves as a universal frame of reference for all local phenomena in space. The Dynamic Universe model discards the space-time marriage, the relativity principle, the Lorentz transformation, the equivalence principle, and dark energy. The DU model also discards the postulation of the constancy of the velocity of light but explains why the velocity of light is observed as being unaffected for observers in motion and for observers at different gravitational potentials.

In celestial mechanics, the Dynamic Universe leads to stable orbits down to the critical radius of local singularities (black holes), shows the perihelion shift of eccentric orbits, the Shapiro delay, the bending of light, etc. In the Dynamic Universe, gravitationally bound systems like planetary systems, galaxies and galaxy groups expand in direct proportion to the expansion of space. As a consequence, the Euclidean appearance of the angular sizes of galaxies is predicted. The DU's prediction for the magnitude versus redshift of standard candles is in an excellent agreement with recent supernova observations without assumptions of dark energy or free parameters.

The Dynamic Universe means a major change in the paradigm but offers a platform – with relativity built in and a firm anchor to human conception – to doctrines like Maxwell's equations and electromagnetism in general, thermodynamics, celestial mechanics, and quantum mechanics.

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1. Introduction

Newtonian physics is local by its nature. No local frame is in a special position in space. There are no overall limits to space or to physical quantities. Newtonian space is Euclidean until infinity, and velocities in space grow linearly as long as there is constant force acting on an object.

Finiteness of physical quantities was first observed about 100 years ago – as finiteness of velocities. The theory of relativity introduces a mathematical structure for the description of the finiteness of velocities by modifying the coordinate quantities, time and distance in such a way that velocities in space never exceed the velocity of light, which was postulated a natural constant. Like in Newtonian physics, no local frame, or inertial observer, is in a special position in space. According to the special and general theories of relativity there is no universal frame of reference in space; velocities in space are studied as velocities relative to an observer or velocities in a local frame of reference. At cosmological distances, the Friedman-Lemaître-Robertson-Walker (FLRW) metric derived from the general theory of relativity predicts expansion of space as a gravitationally bound system as whole but ignores the linkage of the expansion to gravitationally bound

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subsystems in space [1].

In the Dynamic Universe approach [2], space is described as the three-dimensional surface of a four-dimensional sphere that is contracting or expanding in a four-dimensional universe – the dynamics of space being determined by a zero-energy balance of motion and gravitation in the structure. Finiteness of physical quantities in DU space comes from the finiteness of total energy in space. Finiteness of velocities is a consequence of the zero-energy balance, which does not allow velocities higher than the velocity of space in the fourth dimension – in the direction of the radial expansion of the 4-sphere defining space. The velocity of space in the fourth dimension is determined by the zero-energy balance of motion and gravitation of whole space and it serves as the reference for all velocities in space. *Relativity in DU space means relativity of local to the whole — relativity is a measure of the locally available share of the total energy in space.*

There is no space-time linkage in Dynamic Universe; time is universal and the fourth dimension is metric by its nature. The local state of rest is linked – through a chain of nested energy frames – to the state of rest in hypothetical homogeneous space – space as it would be without accumulation of mass into mass centers in space. The rest momentum and the rest energy in a local frame are attributes of the motion and gravitational states of the local frame in its parent frames which are ultimately linked to hypothetical homogenous space serving as the frame of reference for all local frames in space.

Predictions for local phenomena in DU space are essentially the same as the corresponding predictions given by the special and general theories of relativity. At extremes, – at cosmological distances and in the vicinity of local singularities in space – differences in the predictions become meaningful. Reasons for the differences can be traced back to the differences in the basic assumptions and in the structures of the two approaches. Due to the system of nested energy frames and the universal frame of reference there is no need for the relativity principle. The Dynamic Universe relies on the zero-energy principle and the conservation of energy in interactions in space, and discards the equivalence principle. It turns out that for conserving the total energy in space, the buildup of kinetic energy via gravitational acceleration is different from the buildup of kinetic energy via acceleration at constant gravitational potential.

The merits of the DU model are: the DU is a holistic approach relying on absolute time and a universal frame of reference, intelligible basic assumptions, and internal logics with fairly simple mathematics. The DU produces precise, parameter-free predictions both for local phenomena and for cosmological observations in an excellent agreement with observations. The Dynamic Universe allows unified expressions for the rest energy, the energy of electromagnetic radiation, and Coulomb energy. As a measure of locally available rest energy, relativity appears as a natural attribute in quantum mechanical considerations. In Dynamic Universe, Mach's principle gets a quantitative explanation.

2. Energy buildup in space

2.1 Space as the 3-surface of a 4-dimensional sphere

The Dynamic Universe model is primarily an analysis of energy balances in space. Absolute time is postulated and a fourth dimension of metric nature is required for the dynamics of spherically closed 3-dimensional space. Closing space as a 3-dimensional surface of a four-dimensional sphere minimizes the gravitational energy and maximizes the degree of symmetry in the structure. As an initial condition and for calculating the primary balance of the energies of motion and gravitation mass is assumed to be uniformly distributed in space.

> Space as the surface of a 4-sphere is quite an old concept of describing space as a closed but endless entity. Spherically closed space was outlined in the 19th century by Ludwig Schläfli, George Riemann and Ernst Mach. Space as the 3-dimensional surface of a four sphere was also Einstein's original view of the cosmological picture of general relativity in 1917 [3]. The problem, however, was that Einstein was looking for a static solution — it was just to prevent the dynamics of spherically closed space that made Einstein to

add the cosmological constant to the theory. Dynamic space requires metric fourth dimension which does not fit to the concept of four-dimensional spacetime the theory of relativity is relying on.

In his lectures on gravitation in early 1960's Richard Feynman [4] stated: *"...One intriguing suggestion is that the universe has a structure analogous to that of a spherical surface. If we move in any direction on such a surface, we never meet a boundary or end, yet the surface is bounded and finite. It might be that our three-dimensional space is such a thing, a tridimensional surface of a four sphere. The arrangement and distribution of galaxies in the world that we see would then be something analogous to a distribution of spots on a spherical ball."*

In the same lectures [5] Feynman also pondered the equality of the rest energy and gravitational energy in space: *"If now we compare the total gravitational energy $E_g = GM^2_{tot}/R$ to the total rest energy of the universe, $E_{rest} = M_{tot}c^2$, lo and behold, we get the amazing result that $GM^2_{tot}/R = M_{tot}c^2$, so that the total energy of the universe is zero. — It is exciting to think that it costs nothing to create a new particle, since we can create it at the center of the universe where it will have a negative gravitational energy equal to M_Xc^2. — Why this should be so is one of the great mysteries — and therefore one of the important questions of physics. After all, what would be the use of studying physics if the mysteries were not the most important things to investigate."*

Obviously, Feynman did not take into consideration the possibility of a dynamic solution to the "great mystery" of the equality of the rest energy and the gravitational energy in space. In fact, such a solution does not work in framework of relativity theory which is based on time as the fourth dimension.

Starting from the state of rest in homogeneous space with essentially infinite radius means an initial condition where both the energy of motion is zero and the energy of gravitation is zero due to very high distances. Release of gravitational energy – or a trend to minimum potential energy – converts gravitational energy into the energy of motion. Spherically closed space gains motion inward against release of gravitational energy in a contraction phase, and pays back the energy of motion to the energy of gravitation in an expansion phase after passing a singularity. The dynamics of spherically closed space works like that of a spherical pendulum in the fourth dimension as illustrated in Figure 2.1-1.

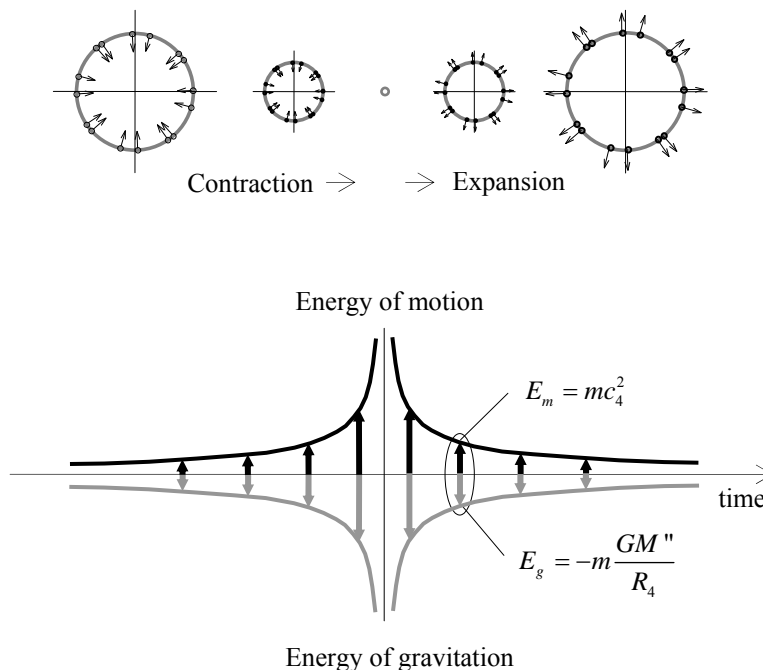Contraction $\Rightarrow$ $\Rightarrow$ Expansion

Figure 2.1-1. Energy buildup and release in spherical space. In the contraction phase, the velocity of motion increases due to the energy gained from the release of gravitational energy. In the expansion phase, the velocity of motion gradually decreases, while the energy of motion gained in contraction is returned to the energy of gravitation.

We define the simplest possible formulas for the energies of motion and gravitation in homogeneous space contracting and expanding in the fourth dimension – Newtonian gravitation for the energy of gravitation of homogeneous mass and the energy of motion as velocity times momentum like we have for the energy of electromagnetic radiation.

The inherent energy of gravitation is defined as

$$E_g = -mG \int_V \frac{\rho dV(r)}{r} \qquad \Rightarrow \qquad E_{g(4)} = -\frac{GM''m}{R_4} \qquad (2.1:1)$$

where $\rho = M_\Sigma/V$ is the mass density in space with volume $V = 2\pi^2 R_4^3$ as the volume of the 3-dimensional "surface" of a 4-dimensional sphere. Release of gravitational energy means the growth of negative energy E_g with a decreasing 4-radius ($E_{g(4)} \rightarrow -\infty$ when $R_4 \rightarrow 0$).

Mass M'' in the second equation (2.1:1) is the mass equivalence of spherically closed space. Any mass m in space is at distance R_4 from mass equivalence $M'' = 0.776 \cdot M_\Sigma$ at the center of the 4-sphere. The factor 0.776 comes from the integral in the first equation (2.1:1).

The inherent energy of motion in the hypothetical environment at rest – as applicable for the contraction and expansion of spherically closed space in the fourth dimension, is defined as

$$E_{m(4)} = c_4 |\mathbf{p}| = c_4 |m\mathbf{c}_4| = mc_4^2 \qquad (2.1:2)$$

where c_4 is used as the notation for the velocity of space in fourth dimension. Interestingly, the format of the inherent energy of motion is equal to Leibnitz's *vis viva*, the very original formulation of kinetic energy.

The zero energy balance of motion and gravitation for mass m in space becomes

$$mc_4^2 - \frac{GmM''}{R_4} = 0 \qquad (2.1:3)$$

and by substitution of the total mass $M_\Sigma = \Sigma m$ for m we get the equation for the zero-energy balance of homogeneous space

$$M_\Sigma c_4^2 - \frac{GM_\Sigma M''}{R_4} = 0 \qquad (2.1:4)$$

Velocity c_4 can be solved from equations (2.1:3) and (2.1:4) in terms of G, M'', and R_4

$$c_4 = \pm \sqrt{\frac{GM''}{R_4}} \qquad \approx 300\,000 \qquad [\text{km/s}] \qquad (2.1:5)$$

The negative value of c_4 in equation (2.1:5) refers to the velocity of contraction and the positive value to the velocity of expansion. The processes of contraction and expansion are symmetrical. All energy of motion gained in the contraction is returned to gravitational energy in the expansion.

The numerical value of $c_4 = 300\,000$ in (2.1:5) is equal to the velocity of light in space. The solution of (2.1:5) is based on total mass obtained from average mass density $\rho = 5 \cdot 10^{-27}$ kg/m^3 and the 4-radius 14 billion light years corresponding to Hubble constant $H_0 \approx 70$ [(km/s)/Mpc]. The mass density of $5 \cdot 10^{-27}$ kg/m^3 is about 0.55 times the Friedman critical mass density used as a reference in FLRW cosmology.

As a consequence of the zero-energy balance in space, the velocity of light in space is equal to the velocity of space in the fourth dimension. This conclusion can be confirmed in a detailed energy analysis or simply by observing that an object moving at $c = c_4$ in space moves at "satellite orbit" around the mass equivalence M'' at the barycenter of spherically closed space.

Due to the dynamic nature of the zero-energy balance in space the velocity of light slows down in the course of the expansion of space. The present annual increase of the R_4 radius of space is $dR_4/R_4 \approx 7.2 \cdot 10^{-11}$/year and the deceleration rate of the expansion is $dc_4/c_4 \approx -3.6 \cdot 10^{-11}$/year, which means also that the velocity of light slows down as $dc/c \approx -3.6 \cdot 10^{-11}$/year. In principle, the

change is large enough to be detected. However, the change is reflected to the ticking frequencies of atomic clocks via the degradation of the rest momentum, i.e. the frequencies of clocks slow down at the same rate as the velocity of light thus disabling the detection.

2.2 The rest energy of matter in homogeneous space

Mass at rest in homogeneous space has momentum $\mathbf{p} = m\mathbf{c}_4$ in the fourth dimension. The associated energy of motion is

$$E_{(rest)m} = c_4 |\mathbf{p}| = c_4 |m\mathbf{c}_4| = mc_4^2 \tag{2.2:1}$$

which means the rest energy of mass m. It is useful to describe the fourth dimension as an imaginary dimension (Figure 2.2-1). The rest energy of mass m is counterbalanced by the global gravitational energy arising from all other mass in space.

There is a complementary linkage between the energies of motion and gravitation in the fourth dimension. The rest energy is a local expression of the energy of a mass object. Being counterbalanced by the global gravitational energy arising from all other mass in space the complementarity of the energies can also be seen as complementarity of local and global, local and all the rest of space. The linkage of local to global provides a basis for understanding Mach's principle, which is discussed later in this paper.

In real space the local velocity of light is affected by the local gravitational environment. On the Earth the local velocity of light, denoted as c, can be estimated as being on the order of parts per million (ppm) lower than the velocity of light in hypothetical homogeneous space, $c < c_0 = c_4$. For generality we rewrite equation (2.2:1) into form

$$E_{(rest)m} = c_4 |\mathbf{p}| = c_0 |m\mathbf{c}| = c_0 mc \tag{2.2:2}$$

which applies in real space.

From equation (2.2:2) we can immediately see that adding momentum $\mathbf{p}$ in a space direction to the rest momentum in the fourth dimension leads to total energy motion

$$E_{m(4)} = c_0 |\mathbf{p}_4 + \mathbf{p}| = c_0 \sqrt{\left(mc_4^2\right) + p^2} \tag{2.2:3}$$

which is essentially the same as the equation for total energy in the special theory of relativity. Importantly, however, equation (2.2:3) does not rely on the relativity principle, the Lorentz transformation, time dilation or any other assumption behind the theory of relativity.
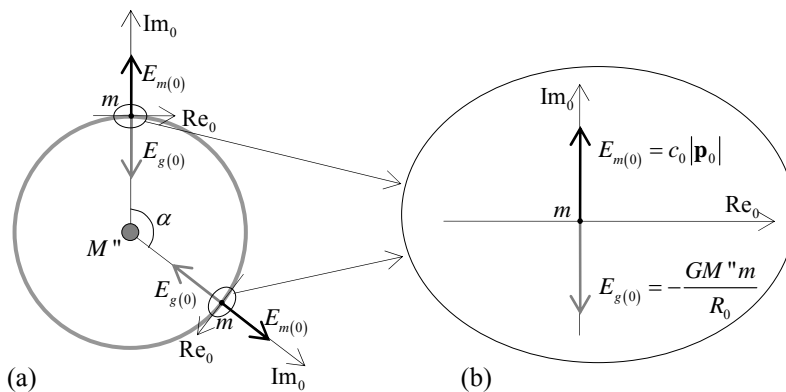

Figure 2.2-1. (a) Hypothetical homogeneous space has the shape of the 3-dimensional "surface" of a perfect 4-dimensional sphere. Mass is uniformly distributed in the structure and the barycenter of mass in space is in the center of the 4-sphere. Mass m is a test mass in hypothetical homogeneous space. (b) In a local presentation a selected space direction is shown as the Re_0 axis, and the fourth dimension which in hypothetical homogeneous space is the direction of R_0 is shown as the Im_0 axis. The velocity of light in hypothetical homogeneous space is equal to the expansion velocity $c_0 = c_4$.

2.3 The energy of electromagnetic radiation and Coulomb energy

Because the velocity of light is not constant, it is important to study the linkage of the velocity of light to other physical quantities considered as constants. A point source of electromagnetic radiation (like an emitting atom) at rest in space moves distance $\Delta L = c/f = \lambda$ in the fourth dimension in a cycle, which allows the study of a point source as a one-wavelength dipole in the fourth dimension [7]. The standard solution of Maxwell's equations for the energy emitted by a dipole of length z_0 in a cycle is

$$E_\lambda = \frac{P}{f} = \frac{N^2 e^2 z_0^2 \mu_0 16\pi^4 f^4}{12\pi cf} = N^2 \left(\frac{z_0}{\lambda}\right)^2 \frac{2}{3}\left(2\pi^3 e^2 \mu_0 c_0\right) f \qquad (2.3:1)$$

The factor 2/3 in (2.3:1) comes from the ratio of average energy density to the energy density in the normal plane of the dipole. The factor N is the number of electrons oscillating in the dipole. For a dipole in the fourth dimension all space directions are perpendicular to the dipole and we may assume factor 2/3 becoming replaced with factor A close to unity. For a point source as a one-wavelength dipole ($z_0 = \lambda$) in the fourth dimension, the energy per electron transition ($N = 1$) in the dipole is

$$E_\lambda = A \cdot 2\pi^3 e^2 \mu_0 c \cdot f = h \cdot f = h_0 c \cdot f = \frac{h_0}{\lambda} c^2 \qquad (2.3:2)$$

where $h_0 \equiv h/c$ is referred to as the intrinsic Planck constant with dimensions [kg/m]. By setting $h = A \cdot 2\pi^3 e^2 \mu_0\, c = 5.99695618 \cdot 10^{-34}$ [CODATA 2006], the value of the factor A is $A = 2.3049$. An important message of equation (2.3:2) is that the velocity of light appears as a hidden factor in the Planck constant. Applying the intrinsic Planck constant, we rewrite equation (2.3:2) into form

$$E_\lambda = \frac{h_0}{\lambda} c_0 c = c_0 |\mathbf{p}| = c_0 \left|\frac{h_0}{\lambda} \mathbf{c}\right| = c_0 m_\lambda c \qquad (2.3:3)$$

where m_λ is referred to as the *mass equivalence of a cycle of radiation* (per a single unit charge transition in a point source). As a requirement of the conservation of total energy, the square of the velocity of light in (2.3:2) shall be divided into the product of c_0 as the velocity of light in hypothetical homogeneous space, and c as the local velocity of light. Obviously, a more general expression for the energy of a cycle of radiation is

$$E_{\lambda(N)} = c_0 |\mathbf{p}| = c_0 \left|N^2 m_\lambda \mathbf{c}\right| = c_0 \cdot N^2 m_\lambda c \qquad (2.3:4)$$

where N^2 serves as an intensity factor. Equations (2.3:3) and (2.3:4) give the energy of a cycle of radiation in the same format as we have for the rest energy of mass. To complete the unification of the expressions of energy, we express the Coulomb energy applying vacuum permeability $\mu_0 = 1/(c_0 c \cdot \varepsilon_0)$ as

$$E_C = N_1 N_2 \frac{e^2 \mu_0}{2\pi r} c_0 c = N_1 N_2 \alpha \frac{h_0}{2\pi r} c_0 c = c_0 m_C c \quad ; \quad \Delta E_C = c_0 c \cdot \Delta m_C \qquad (2.3:5)$$

where m_C is the mass equivalence of Coulomb energy and α is the fine structure constant

$$\alpha = \frac{e^2 \mu_0}{2 h_0} = \frac{e^2 \mu_0}{2 \cdot 1.1049 \cdot 2\pi^3 e^2 \mu_0} = \frac{1}{1.1049 \cdot 4\pi^3} \approx \frac{1}{137.036} \qquad (2.3:6)$$

As illustrated by the last part of (2.3:5), the release of Coulomb energy by changing the distance of charges can be expressed in terms of release of mass equivalence Δm_C [kg].

Interestingly, applying the breakdown of the Planck constant obtained from Maxwell's equation, the fine structure constant α appears as a purely numerical factor without linkage to any physical constant.

2.4 Unified expression of energy

Comparison of equations (2.2:2), (2.3:4), and (2.3:5) shows the unity between the rest energy of mass and the electromagnetic energies. Equations (2.3:3) and (2.3:4) express electromagnetic energies in terms of mass equivalences. Correspondingly, the rest energy of a mass object can be expressed in terms of wavelength equivalence λ_m or wave number equivalence k_m ($=2\pi/\lambda$) of mass as

$$E_{(rest)m} = c_0 mc = \frac{h_0}{\lambda_m} c_0 c = \hbar_0 k_m \cdot c_0 c \qquad (2.4:1)$$

The wavelength equivalence of mass in (2.4:1) is equal to the Compton wavelength and the wave number equivalence equal to the Compton wave number of mass m ($\hbar_0 = h_0/2\pi$), Figure 2.4-1.

In the Dynamic Universe the velocity of light changes both locally and in the course of the expansion of space. Mass (including the mass equivalences of electromagnetic energy) is postulated as the main invariant throughout the process of contraction and expansion of space. The unified expression of energy allows a detailed study of the conservation of energy in interactions in space by separating the contributions of mass as the substance for the expression of energy and the velocity of light.

Expression of energies in the Dynamic Universe is complementary. The energy of motion is obtained against release of a potential energy. In the energy book-keeping the potential energy released serves as the negative counterpart to the positive energy of motion gained. In the DU framework the rest energy of matter is the local expression of energy which is counterbalanced by the global energy of gravitation due to the rest of mass in space.

The linkage of local and global is a characteristic feature of the Dynamic Universe. The whole in the Dynamic Universe is not considered as the sum of elementary units — the multiplicity of elementary units is considered as the result of diversification of whole. There are no independent objects in space — everything is linked to the rest of space with a certain likeness to the Leibniz's monad [6].

Coulomb energy (2.5:5)

$$E_C = N_1 N_2 \frac{e^2 \mu_0}{2\pi r} c_0 c = N_1 N_2 \alpha \frac{h_0}{2\pi r} c_0 c = N_1 N_2 m_C \cdot c_0 c$$

Energy of a cycle of electromagnetic radiation (2.3:4)

$$E_{\lambda(N)} = c_0 |\mathbf{p}| = c_0 \left| N^2 m_\lambda \mathbf{c} \right| = N^2 m_\lambda \cdot c_0 c$$

Rest energy of localized energy object (2.2:1), (2.4:1)

$$E_{(rest)m} = c_0 |\mathbf{p}_4| = m \cdot c_0 c = \frac{h_0}{\lambda_m} c_0 c = \hbar_0 k_m \cdot c_0 c$$
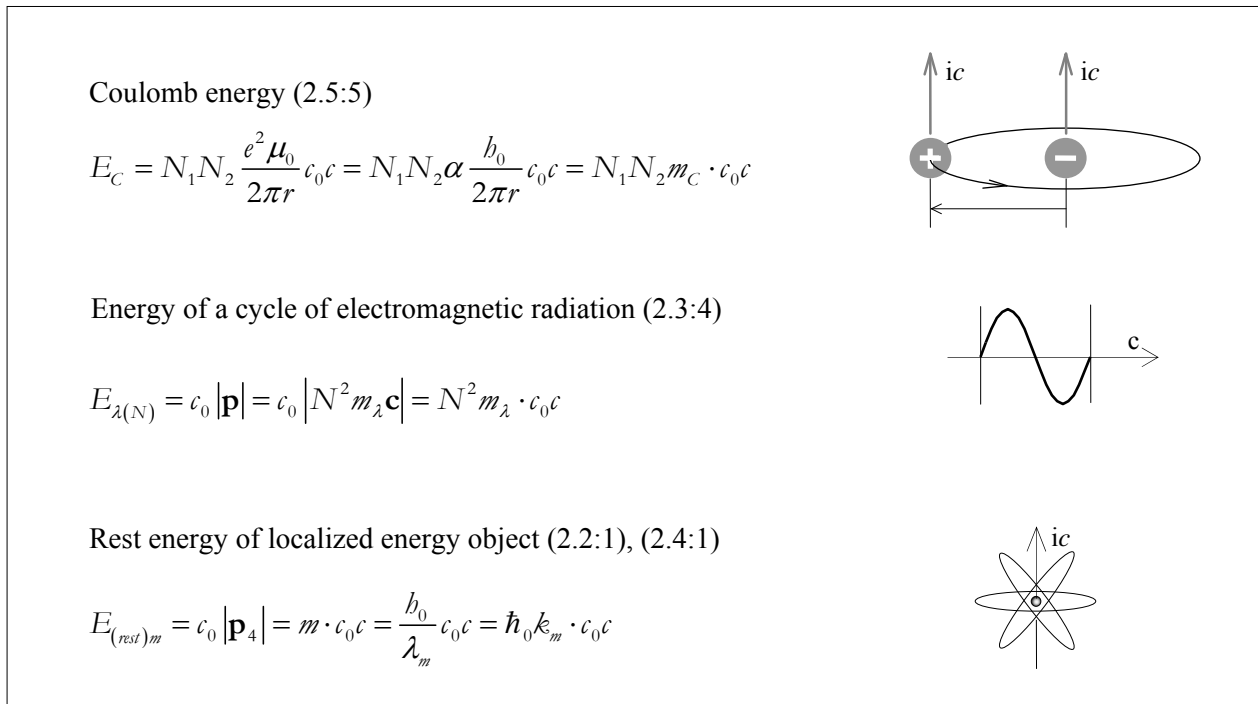
Figure 2.4-1. Unified expressions for the Coulomb energy, the energy of a cycle of electromagnetic radiation, and the rest energy of a localized mass object.

2.5 Main postulates in the Dynamic Universe

We can summarize the main postulates of the Dynamic Universe as follows:

1   Space is defined as the three-dimensional surface of a four-dimensional sphere free to contract and expand in an infinite four-dimensional universe.

2   Time is a universal scalar. The fourth dimension is metric by its nature.

3   As the initial condition, all mass is homogeneously distributed in space. Total mass in space is conserved in all interactions in space.

4   The dynamics of space is determined by a zero-energy balance of motion and gravitation in the structure.

5   Inherent energy of gravitation is defined in hypothetical homogeneous space; inherent energy of motion is defined in the hypothetical environment at rest.

6   The buildup of motion, electromagnetic energy, elementary particles, and mass centers *within space* conserves the total energy and the zero-energy balance created in the contraction–expansion process *of space*.

Classically, *force and field* are primary quantities and energy is derived by integration of force. The prevailing cosmological appearance and structure of space is an extrapolation of local spacetime based on equivalence principle equating inertial and gravitational accelerations.

The postulates in the DU reflect the holistic approach. The total structure of space and the *inherent expressions of energy* are postulated; force is derived as the gradient of energy. Local structures are derived from the whole by conserving the total energy in space.

The DU approach discards the central postulates of the prevailing theories:

1.   The velocity of light is not postulated as a constant or an invariant.

2.   Time is not regarded as a fourth dimension; the space-time concept is ignored.

3.   There is no postulated equation of motion or force/acceleration.

4.   There is no need or basis for equivalence principle.

5.   There is no need or basis for Lorentz transformation.

6.   There is no need or basis for relativity principle.

7.   There is no need or basis for dark energy.

In local considerations it is useful to describe the fourth dimension as an imaginary direction; local momentums and energies are presented as complex functions with imaginary parts showing the effects in the fourth dimension. In such a formalism, e.g. the rest energy of matter appears as the imaginary component of the energy of motion. A mass object moving in space has both imaginary and real components in the total energy of motion. In the DU framework, the absolute values of the complex energies are equivalent to the corresponding energies in the prevailing formalism where energy is used as a scalar (real) function only.

The use of complex functions is a powerful tool for a detailed analysis of energy structures in space. In spherically closed space the imaginary direction (the fourth dimension) is a kind of sum of symmetry in three space directions, e.g. the global gravitational energy arising equally from all space directions is equivalent to gravitational energy arising from the barycenter of space in the fourth dimension.

The breakdown of Planck constant and the identification of the intrinsic Planck constant is an exceedingly important step for the unified expression of energies and for understanding the wave-like nature of mass as the substance for all expressions of energy. Applying the intrinsic Planck constant mass can be expressed in terms of a wavelength equivalence or wave number equivalence. A mass object moving at velocity $\beta = v/c$ in space can be described in terms of a wave structure in four dimensions by rewriting the energy-momentum four-vector

$$E^2 = \left(mc^2\right)^2 + \left(pc\right)^2 \tag{2.5:1}$$

into form

$$c_0^2 \cdot \hbar_0^2 k_{(m+\Delta m)}^2 c^2 = c_0^2 \cdot \hbar_0^2 k_{(m+\Delta m)}^2 \beta^2 c^2 + c_0^2 \cdot \hbar_0 k_{(m)}^2 c^2 \tag{2.5:2}$$

and further

$$k_{(m+\Delta m)}^2 = \beta^2 k_{(m+\Delta m)}^2 + k_{(m)}^2 \tag{2.5:3}$$

or as complex wave structure

$$k_{(m+\Delta m)}^* = \beta k_{(m+\Delta m)} + \mathrm{i}\, k_{(m)} \tag{2.5:4}$$

where (*) is used as the notation for a complex function.

In the DU framework Planck mass and Planck distance obtain the forms

Planck mass $\qquad m_0 = c_0 \sqrt{\dfrac{h_0}{G}} = 5.4556 \cdot 10^{-8} \qquad$ [kg] $\tag{2.5:5}$

Planck distance $\qquad r_0 = \dfrac{1}{c_0} \sqrt{h_0 G} = 4.05 \cdot 10^{-35} \qquad$ [m] $\tag{2.5:6}$

which means that the Planck distance is the wavelength equivalence of the Planck mass

$$r_0 = \lambda_{(m_0)} = \frac{h_0}{m_0} \qquad ; \qquad m_0 = \frac{h_0}{\lambda_{(m_0)}} = \hbar_0 k_{(m_0)} \tag{2.5:7}$$

Equation (2.5:7) is of special interest as the basis for the buildup of elementary particles as sub-harmonics of the wavelength equivalence of Planck mass as suggested by Ari Lehto [8].

Combining of equations (2.5:5) and (2.5:6) with (2.1:4) relates the Planck mass and the Planck distance to the mass equivalence M'' and the 4-radius R_4 of space as

$$m_0 = \frac{r_0}{R_4} M'' = 3.1 \cdot 10^{-61} M'' \; ; \qquad r_0 = \frac{m_0}{M''} R_4 = 3.1 \cdot 10^{-61} R_4 \tag{2.5:8}$$

respectively.

3. From homogeneous space to real space

3.1 The buildup of mass-centers; free fall in space

The basic zero-energy balance in space was calculated in hypothetical homogeneous space with all mass homogeneously in the "surface" of an ideal 4-sphere. The first step towards real space is the buildup of mass centers in space. Mass at rest in homogeneous has momentum in the fourth dimension only. Conservation of the energies of motion and gravitation requires local tilting of space in a mass center buildup. Tilting of space means a turn of the local fourth dimension (denoted as imaginary direction) which creates buildup of the momentum of free fall in space as an orthogonal component to the reduced momentum in the local fourth dimension Figure 3.1-1.

Applying the complex number presentation, with the imaginary axis as the fourth dimension and the real axis as a space direction, the total energy of motion of a test mass m in free fall in tilted space (Im_ϕ, Re_ϕ) is
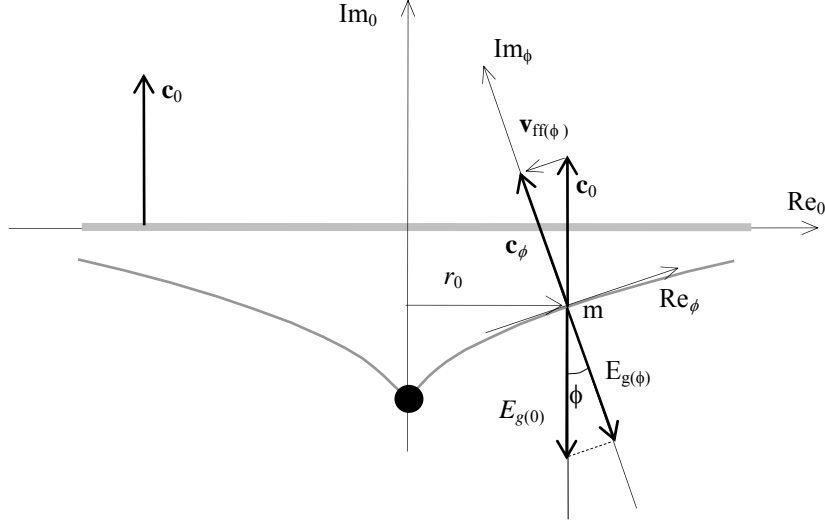
Figure 3.1-1. As a consequence of the conservation of the primary energies of motion and gravitation, the buildup of a mass center in space bends the spherical space locally causing a tilting of space near the mass center. The local imaginary axis is always perpendicular to local space. As a consequence, the local imaginary velocity of space, and also the local velocity of light are reduced in tilted space.

$$E_{m(\phi),total} = c_0 \left| \mathbf{p}_0 \right| = c_0 \left| \mathbf{p}_{\mathrm{Im}(\phi)} + \mathbf{p}_{\mathrm{Re}(\phi)} \right| = c_0 \left| \mathbf{p}_{rest(\phi)} + \mathbf{p}_{ff(\phi)} \right| \tag{3.1:1}$$

The velocity of free fall is obtained against a reduction in the imaginary velocity of space and the buildup of kinetic energy in free fall is achieved against a reduction of the local rest energy

$$E_{kin(ff)} = c_0 \Delta \left| \mathbf{p} \right| = c_0 \left(\left| \mathbf{p}_{rest(0)} \right| - \left| \mathbf{p}_{rest(\phi)} \right| \right) = c_0 \left(\left| m\mathbf{c}_0 \right| - \left| m\mathbf{c}_\phi \right| \right) = c_0 m \, \Delta c \tag{3.1:2}$$

where the local velocity of light, which is equal to the velocity of space in the local fourth dimension, is denoted as c ($c = c_\phi$). The reduction of the global gravitational energy in tilted space is equal to the gravitational energy removed from the spherical symmetry in homogeneous space

$$E_{g(\mathrm{Im}\phi)} = E_{g(\mathrm{Im}_0)} \left(1 - \delta \right) \tag{3.1:3}$$

where δ is denoted as the local gravitational factor (=local gravitational energy/total gravitational energy)

$$\delta = \frac{GM}{r_0} \Big/ \frac{GM''}{R_4} = \frac{GM}{c_0^2 r_0} = 1 - \cos\phi \tag{3.1:4}$$

where r_0 is the distance of m from the local mass center M in the direction of non-tilted space. The tilting of local space in the vicinity of a local mass center reduces the local velocity of light as

$$c_{local} = c = c_0 \cos\phi = c_0 \left(1 - \delta \right) \tag{3.1:5}$$

The reduction of the local velocity of light together with the increased distance along the dent in the vicinity of a mass center in space is observed as the Shapiro delay and the deflection of light passing a mass center in space.

In real space mass center buildup occurs in several steps leading to a system of nested gravitational frames, Fig. 3.1-2.
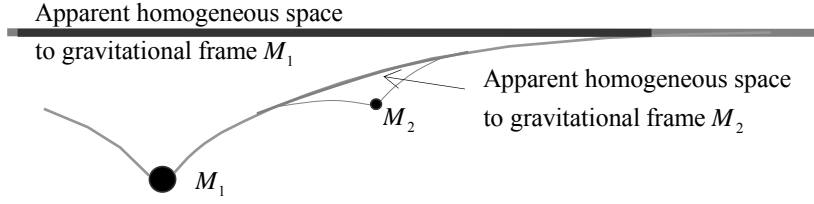
Figure 3.1-2. Hypothetical non-tilted space in the vicinity of a local mass center is referred to as apparent homogeneous space to the local gravitational frame. Accumulation of mass into mass centers to form local gravitational frames occurs in several steps. Starting from hypothetical homogeneous space, the "first-order" gravitational frames, like M_1 in the figure, have hypothetical homogeneous space as the parent frame. In subsequent steps, smaller mass centers may be formed within the tilted space around in the "first order" frames. For those frames, like M_2 in the figure, space in the M_1 frame, as it would be without the mass center M_2, serves as the parent frame and the apparent homogeneous space to frame M_2.

For each gravitational frame the surrounding space appears as apparent homogeneous space which serves as the parent frame and the closest reference to the global gravitational energy and the velocity of light in the local frame. Through the system of nested gravitational frames the local velocity of light is related to the velocity of light in hypothetical homogeneous space as

$$c_n = c = c_0 \prod_{i=1}^{n} \cos\phi = c_0 \prod_{i=1}^{n} (1 - \delta_i) \qquad (3.1:6)$$

The momentum of an object at rest in a gravitational state is the rest momentum in the direction of the local fourth dimension, the local imaginary direction.

3.2 Motion at constant gravitational potential

In free fall velocity in space is obtained against a reduction in the local velocity of light via tilting of space. Kinetic energy in free fall is obtained against reduction in the rest energy of the falling object. Mass in free fall is conserved and the total energy is conserved.

Buildup of kinetic energy at constant gravitational potential requires additional energy from a local source. Acceleration of a charged object in a Coulomb field is expressed in terms of a release of Coulomb mass equivalence Δm_C in equation (2.3:5). The total energy of an object accelerated in Coulomb field in space receives Coulomb energy $\Delta E_C = \Delta m_C \cdot c_0 c$ to be converted into kinetic energy E_{kin}, [Figure 3.2-1(a)]

$$E_{m(tot)} = E_{rest(0)} + \Delta E_{Coulomb} = E_{rest(0)} + \Delta m \cdot c_0 c = E_{rest(0)} + E_{kin} = c_0 \left(m + \Delta m \right) c \qquad (3.2:1)$$

A complex presentation illustrates the buildup of the momentum and the total energy of motion as the orthogonal sum of the momentum at rest in the imaginary direction and the momentum created in space

$$E_{m(tot)} = c_0 c \left(m + \Delta m \right) = c_0 \left| \mathbf{p} * \right| = c_0 \left| \mathbf{p}' + \mathbf{i}\, \mathbf{p}_0 '' \right|$$

$$= c_0 \left| \left(m + \Delta m \right) \mathbf{v} + \mathbf{i}\, mc \right| = c_0 \sqrt{\left(mc \right)^2 + \left(m + \Delta m \right)^2 v^2} \qquad (3.2:2)$$

where the notation (*) relates to a complex function, and (') to the real part and (") to the imaginary part of a complex function.

Combining equations (3.2:1) and (3.2:2) we can relate the increased mass $(m+\Delta m)$ to velocity $\beta = v/c$ as
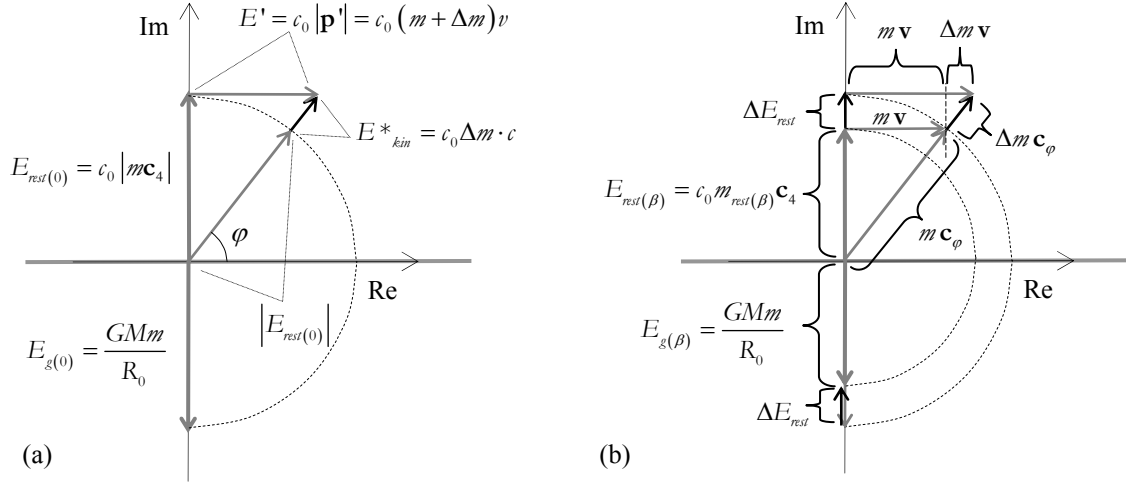
Figure 3.2-1. Kinetic energy and momentum by insertion of energy $\Delta E = c_0 \Delta m \cdot c$ at constant gravitational potential. (a) Components of complex energy. (b) Components of momentum in the direction of the real and imaginary axes and in the direction of total momentum.

$$m_\beta = \frac{m}{\sqrt{1-\beta^2}} \qquad (3.2{:}3)$$

which is equal to the expression of the relativistic mass in special relativity.

Relativistic mass is not a consequence of the velocity in space but it is the mass contribution needed to build up velocity at constant gravitational potential – by conserving the total energy in space.

As illustrated in Figure 3.2-1(b) the momentum in space, the real component of the total momentum, is built up from two parts

$$\mathbf{p}' = (m + \Delta m)\mathbf{v} = \mathbf{p}'_0 + \mathbf{p}'_\Delta = m \cdot \mathbf{v} + \Delta m \cdot \mathbf{v} \qquad (3.2{:}4)$$

Part $\mathbf{p}'_0 = m\mathbf{v}$ can be identified as the real component of the *internal momentum* $\mathbf{p}_{I(\varphi)} = m\mathbf{c}_\varphi$, which is the contribution of the rest momentum to the total momentum $\mathbf{p}_{total(\varphi)}$. Part $\mathbf{p}_\Delta = \Delta m\mathbf{v}$ in the momentum in space results from the mass equivalence released by the Coulomb energy. As a consequence, the imaginary part of the internal momentum,

$$\mathrm{Im}\{\mathbf{p}_{I(\varphi)}\} = m\mathbf{c}_4\sqrt{1-\beta^2} = m_{rest(\beta)}\mathbf{c}_4 = \mathbf{p}_{rest(\beta)} \qquad (3.2{:}5)$$

is the rest momentum of an object moving at velocity $\mathbf{v}$ ($\beta = v/c$) in space. A reduction of the rest momentum means reduction of the rest mass because the velocity in the fourth dimension is fixed to the velocity of space. The reduction of the rest mass affects the imaginary energies of motion and gravitation of an object moving in space equally.

The imaginary part of the kinetic energy $\mathrm{Im}\{E_{kin}\} = \Delta E_{rest}$ is the work done for reducing the global gravitational energy of the object in motion [see Figure 3.2-1(b)].

The reduction of the rest mass and the related reduction of the rest energy and the global gravitational energy can be understood as a quantitative expression of Mach's principle.

The reduction of the rest momentum can also be understood as a consequence of the central force created by motion in spherical space in the fourth dimension. The conclusion resulting from an energy analysis and from an analysis of the central force approach is exactly the same:

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The zero-energy balance of motion and gravitation in the fourth dimension is obtained equally

for mass m_β $\left(= m / \sqrt{1-\beta^2}\right)$ moving at velocity β in space $\qquad$ (3.2:6)

and

for mass $m_{rest(\beta)}$ $\left(= m\sqrt{1-\beta^2}\right)$ at rest in space $\qquad$ (3.2:7)

which means that mass $m_{rest(\beta)}$ serves as the rest mass for phenomena within a moving frame – and allows the definition of the local state of rest.

A local state of rest is bought at the cost of locally available rest energy. As a practical consequence, clocks in a moving frame run slower than clocks at rest in the parent frame.

3.3 The system of nested energy frames

The rest energy of an object m moving at velocity β_n in frame n is

$$E_{rest(n),\beta} = c_0 c \cdot m_{rest(n),0} \sqrt{1-\beta_n^2} \qquad (3.3:1)$$

where $m_{rest(n),0}$ is the rest mass of the object at rest in frame n.

When the whole frame n is in motion at velocity $\beta_{(n-1)}$ in frame $(n-1)$ which is the parent frame to frame n, rest mass $m_{rest(n),0}$ can be related to the rest mass of the object at rest in frame $(n-1)$ as

$$m_{rest(n),0} = m_{rest(n-1),0} \sqrt{1-\beta_{n-1}^2} \qquad (3.3:2)$$

where $m_{rest(n-1),0}$ is the rest mass of the object at rest in frame $(n-1)$.

In a system of n nested frames the rest energy in the n:th frame can be expressed

$$E_{rest(n)} = c_0 mc \prod_{i=1}^{n} \sqrt{1-\beta_i^2} \qquad (3.3:3)$$

Substitution of equation (3.1:6) for c in (3.3:3) adds the effect of the gravitational state of each frame in its parent frame

$$E_{rest(n)} = c_0 mc_0 \prod_{i=1}^{n} (1-\delta_i) \sqrt{1-\beta_i^2} \qquad (3.3:4)$$

Equation (3.3:4) relates the locally available rest energy of an object in the n:th energy frame to rest energy the object had at rest in hypothetical homogeneous space. The local rest energy is affected

- by gravitation in each frame as a reduction of the local velocity of light and
- by motion in each frame as a reduction of the locally available rest mass.

On the Earth, in the Earth gravitational frame we are subject to the effects of the gravitation and rotation of the Earth, the gravitational state and velocity of the Earth in the solar frame, the gravitational state and velocity of the solar system in the Milky Way frame, the gravitational state and velocity of the Milky Way galaxy in the Local Group, and the gravitational state and velocity of the local group in hypothetical homogeneous space which may be presented by the Cosmic Microwave Background as the universal reference at rest, Figure 3.3-1.
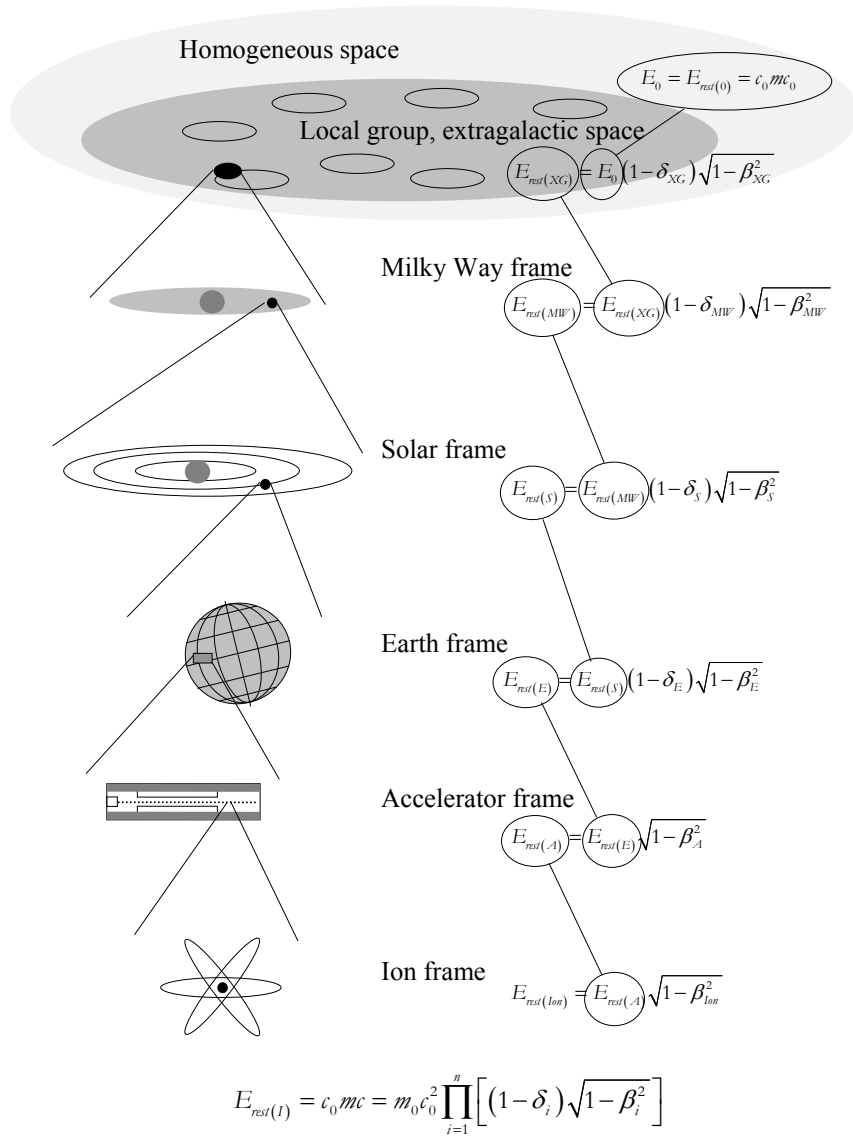
Figure 3.3-1. The rest energy of an object in a local frame is linked to the rest energy of the local frame in its parent frame. The system of nested energy frames relates the rest energy of an object in a local frame to the rest energy of the object in homogeneous space.

3.4 Characteristic emission and absorption frequencies

Application of equation (3.3:4) for the rest energy of an electron, results in the energy states in the standard solution of atoms becoming functions of the gravitational state and motion of the atom studied. When inserted into Balmer's equation the characteristic emission and absorption frequencies become functions of the gravitational state and velocity of the emitting or absorbing atom

$$f_{(n1,n2)} = \frac{\Delta E_{(n1,n2)}}{h_0 c_0} = f_{0(n1,n2)} \prod_{i=1}^{n} (1-\delta_i)\sqrt{1-\beta_i^2} \qquad (3.4:1)$$

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Using the frequency $f_{0,0}$ at rest in apparent homogeneous space of the local frame as the reference, the local characteristic frequency can be expressed

$$f_{\delta,\beta(DU)} = f_{0,0}\left(1-\delta\right)\sqrt{1-\beta^2} \approx f_{0,0}\left(1-\delta-\frac{1}{2}\beta^2-\frac{1}{8}\beta^4+\frac{1}{2}\delta\beta^2\right) \tag{3.4:2}$$

In general relativity, the combined effect of motion and gravitation on the "proper frequency" of atomic oscillators in a local (Schwarzschild) gravitational frame is given by equation

$$f_{\delta,\beta(GR)} = f_{0,0}\sqrt{1-2\delta-\beta^2} \approx f_{0,0}\left(1-\delta-\frac{1}{2}\beta^2-\frac{1}{8}\beta^4-\frac{1}{2}\delta\beta^2-\frac{1}{2}\delta^2\right) \tag{3.4:3}$$

On Earth and in near space conditions the difference in the frequencies given by equations (3.4:2) and (3.4:3) appears in the 18th decimal at most, which is too small a difference to be detected.

The DU predictions (3.4:1) and (3.4:2) do not rely on any assumptions of the relativity theory but are just consequences of the conservation of total energy in spherically closed space.

3.5 Electromagnetic radiation observed in a moving frame

In the DU framework the Doppler effect of electromagnetic radiation can be derived following the classical procedure taking into account both the velocity of the source and the receiver in a frame in common to the source and the receiver. By adding the effect of the velocity of the source on the transmitted frequency and the velocity of the receiver on the frequency of a reference oscillator moving with the receiver (3.4:2) we get

$$f_{A(B)} = f_B\frac{\left(1-\delta_A\right)\sqrt{1-\beta_A^2}}{\left(1-\delta_B\right)\sqrt{1-\beta_B^2}}\frac{\left(1-\beta_{B(\mathbf{r})}\right)}{\left(1-\beta_{A(\mathbf{r})}\right)} = f_A\frac{\left(1-\beta_{B(\mathbf{r})}\right)}{\left(1-\beta_{A(\mathbf{r})}\right)} \tag{3.5:1}$$

where δ_A and β_A are the gravitational factor and velocity of the source, and δ_B and β_B are the gravitational factor and velocity of the receiver. Velocities $\beta_{A(r)}$ and $\beta_{B(r)}$ are the velocity components of the source and the receiver in the direction of the signal path. Equation (3.5:1) is essentially the same as the equation for Doppler effect in the general theory of relativity.

Let's assume that the radiation source is at rest in a propagation frame, i.e. $\beta_{A(r)} = 0$ in equation (3.5:1). The momentum of Doppler shifted radiation observed at B moving at velocity $\beta_{B(r)}$ is

$$\mathbf{p}_B = \mathbf{p}_0\left(1-\beta_{B(\mathbf{r})}\right) = h_0 f_0\left(1-\beta_{B(\mathbf{r})}\right)\hat{\mathbf{r}} = \frac{h_0}{\lambda_0}\left(1-\beta_{B(\mathbf{r})}\right)\mathbf{c}_0 \tag{3.5:2}$$

where h_0 is the intrinsic Planck constant defined in (2.3:2). As shown in equation (3.5:2) both the wavelength and frequency of the Doppler shifted radiation observed at B are changed compared to the wavelength and frequency measured at rest in the propagation frame. As a result, the phase velocity of radiation observed in frame B moving at velocity $\beta_{B(r)}$ is

$$c_B = f_B\lambda_B = f_0\left(1-\beta_{B(\mathbf{r})}\right)\cdot\frac{\lambda_0}{\left(1-\beta_{B(\mathbf{r})}\right)} = f_0\lambda_0 = c_0 \tag{3.5:3}$$

i.e. the phase velocity of radiation observed in the moving frame is equal to the phase velocity of radiation in the rest frame (propagation frame).

If the source is taken to the same moving frame with the receiver there is no change in the momentum of radiation between the source and the receiver even if we think that the propagation of radiation occurs in the underlying rest frame. Such a situation occurs in Michelson–Morley

interferometer; *conservation of the momentum of radiation in the interferometer frame guaranties zero-result in Michelson–Morley experiment.*

When a mass object with momentum $\mathbf{p}_0 = m\mathbf{v}_0$ in a rest frame is taken to a frame moving at velocity $\mathbf{v}_B$ ($\mathbf{v}_0 \parallel \mathbf{v}_B$) momentum $\mathbf{p}_0$, as observed in the moving frame, is reduced by receiver's velocity as

$$\mathbf{p}_B = \mathbf{p}_0 \left(1 - \frac{v_B}{v_0} \right) = \mathbf{p}_0 \left(1 - \beta_B \right) \qquad (3.5{:}4)$$

where $\beta_B = v_B/v_0$. Comparison of (3.5:4) with (3.5:2) shows that the reduction of momentum of radiation and a mass object is equal, but:

In the case of a mass object the change in momentum is observed as change in velocity. In the case of radiation the change in momentum is observed as a change in the wavelength (or mass equivalence of radiation, $m_\lambda = h_0/\lambda$).

3.6 Gravitational shift of clock frequency and radiation

The frequency of an atomic oscillator is directly proportional to the local velocity of light. Both the velocity of light and the frequency of an atomic oscillator are functions of the gravitational state as generally expressed in (3.4:1) and in a local gravitational frame in (1.3:2). Accordingly, the wavelength emitted by an atomic oscillator is independent of the gravitational state. As required by absolute time, the frequency of radiation is conserved when transmitted from a gravitational state to another (same number of cycles as sent in a time interval is received). When observed on ground, the wavelength of radiation emitted from altitude h is shortened (blue-shifted) as

$$\lambda_{A(B)} = \frac{c_B}{c_A}\lambda_A = \frac{c_B}{c_A}\lambda_B \qquad (3.6{:}1)$$

where λ_B is the wavelength emitted by a reference oscillator on the ground.

The gravitational blueshift of clocks results from the effect of gravitation on the frequency of the clocks – the gravitational blueshift of radiation comes from the effect of gravitation on the local velocity of light.

4. Celestial mechanics and cosmology

4.1 Gravitation in Schwarzschild space and in DU space

Table 3.2-I summarizes some predictions related to celestial mechanics in Schwarzschild space which is the GR counterpart of the DU space in the vicinity of a local mass center in space. At a low gravitational field, far from a mass center, the velocities of free fall as well as the orbital velocities in Schwarzschild space and DU space are essentially the same as the corresponding Newtonian velocities. Close to the critical radius, however, differences become meaningful.

In Schwarzschild space the critical radius is

$$r_{c(Schwd)} = \frac{2GM}{c^2} \qquad (4.1{:}1)$$

which is the radius where Newtonian free fall from infinity achieves the velocity of light. The critical radius in DU space is

$$r_{c(DU)} = \frac{GM}{c_0 c_{0\delta}} \approx \frac{GM}{c^2} \qquad (4.1{:}2)$$

	Local relativity (Schwarzschild space [9])	Global relativity (DU space)
1) Velocity of free fall $\left(\delta = GM/rc^2\right)$	$\beta_{ff} = \sqrt{2\delta}\left(1 - 2\delta\right)$ (coordinate velocity)	$\beta_{ff} = \sqrt{1/\left(1-\delta\right)^2 - 1}$
2) Orbital velocity at circular orbits	$\beta_{orb} = \dfrac{1 - 2\delta}{\sqrt{1/\delta - 3}}$ (coordinate velocity)	$\beta_{orb} = \sqrt{\delta\left(1-\delta\right)^3}$
3) Orbital period in Schwarzschild space (coordinate period) and in DU space	$P = \dfrac{2\pi r}{c}\sqrt{\dfrac{2}{\delta}}\quad(=P_{Newton})$ $r > 3 \cdot r_{c(Schwd)}$	$P = \dfrac{2\pi r_c}{c_{0\delta}}\left[\delta\left(1-\delta\right)\right]^{-3/2}$
4) Perihelion advance for a full revolution	$\Delta\psi\left(2\pi\right) = \dfrac{6\pi G\left(M+m\right)}{c^2 a\left(1-e^2\right)}$	$\Delta\psi\left(2\pi\right) = \dfrac{6\pi G\left(M+m\right)}{c^2 a\left(1-e^2\right)}$

Table 4.1-I. Comparison of predictions related to celestial mechanics in Schwarzschild space and in DU space.

which is half of the critical radius in Schwarzschild space. The two different velocities c_0 and $c_{0\delta}$ in (3.2:2) are the velocity of hypothetical homogeneous space and the velocity of apparent homogeneous space in the fourth dimension.
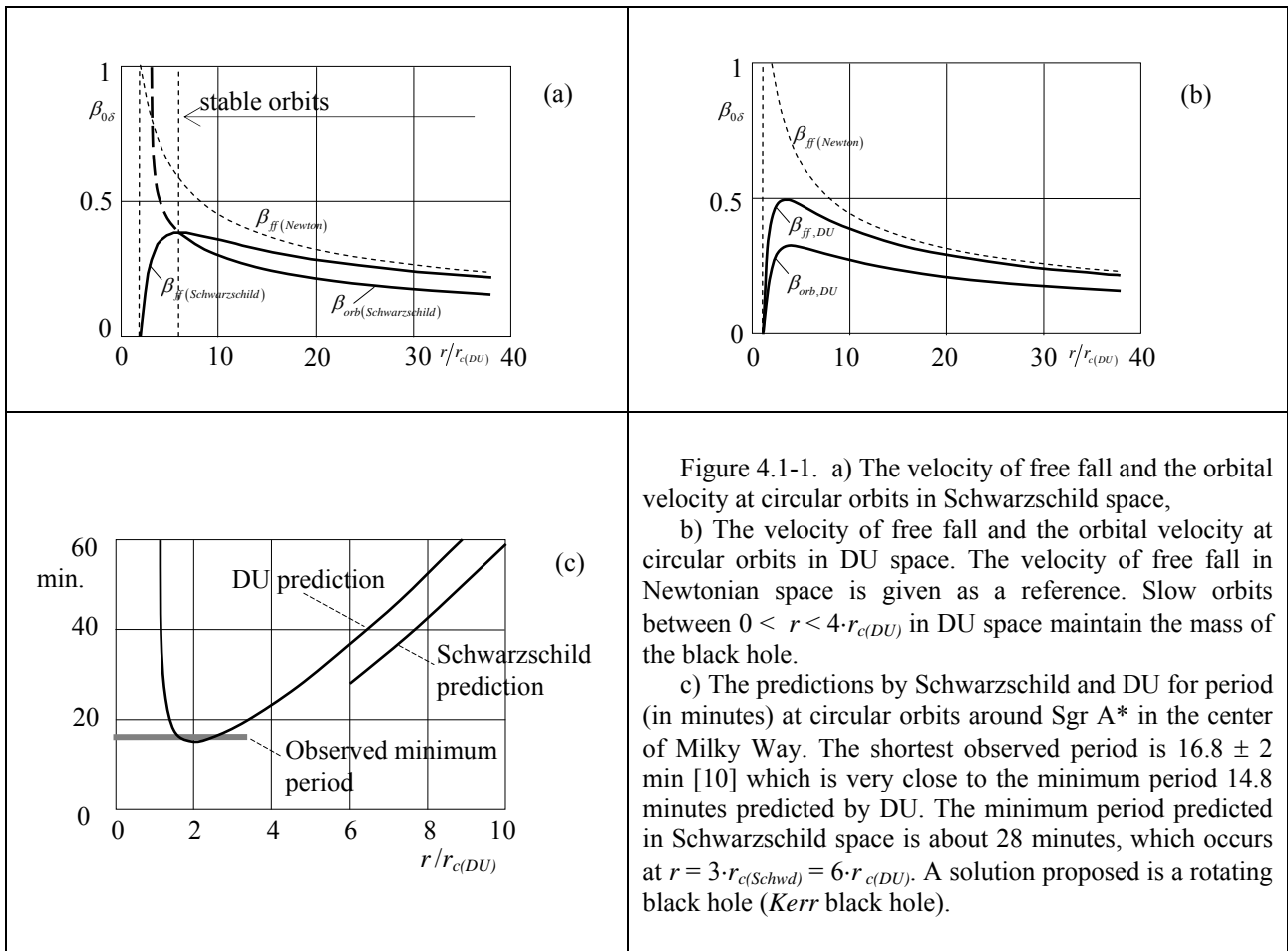

Figure 4.1-1. a) The velocity of free fall and the orbital velocity at circular orbits in Schwarzschild space,

b) The velocity of free fall and the orbital velocity at circular orbits in DU space. The velocity of free fall in Newtonian space is given as a reference. Slow orbits between $0 < r < 4 \cdot r_{c(DU)}$ in DU space maintain the mass of the black hole.

c) The predictions by Schwarzschild and DU for period (in minutes) at circular orbits around Sgr A* in the center of Milky Way. The shortest observed period is 16.8 ± 2 min [10] which is very close to the minimum period 14.8 minutes predicted by DU. The minimum period predicted in Schwarzschild space is about 28 minutes, which occurs at $r = 3 \cdot r_{c(Schwd)} = 6 \cdot r_{c(DU)}$. A solution proposed is a rotating black hole (Kerr black hole).

In Schwarzschild space the predicted orbital velocity at circular orbit exceeds the velocity of free fall when r is smaller than 3 times the Schwarzschild critical radius, which makes stable orbits impossible. In DU space orbital velocity decreases smoothly towards zero at $r = r_{c(DU)}$, which means that there are stable slow orbits between $0 < r < 4 \cdot r_{c(DU)}$, Fig. 4.1-1(a,b).

The importance of the slow orbits near the critical radius is that they maintain the mass of the black hole.

The prediction for the orbital period at circular orbits in Schwarzschild space applies only for radii $r > 3 \cdot r_{c(Schwd)}$. The black hole at the center of the Milky Way, the compact radio source Sgr A*, has the estimated mass of about 3.6 times the solar mass which means $M_{black\ hole} \approx 7.2 \cdot 10^{36}$ kg, which gives a period of 28 minutes at the minimum stable radius $r = 3 \cdot r_{c(Schwd)}$ in Schwarzschild space. The shortest observed period at Sgr A* is 16.8 ± 2 min [10] which is very close to the prediction of minimum period 14.8 min in DU space at $r = 2 \cdot r_{c(DU)}$, Fig. 4.1-1(c).

The predictions for the perihelion advance in elliptic orbits are essentially the same in Schwarzschild space and in DU space. In DU space the prediction can be derived in a closed mathematical form.

4.2 Cosmological appearance of DU space

The precise geometry, absolute time, and the linkage of the velocity of light to the velocity of the expansion of space along the 4-radius allow a parameter-free derivation of primary cosmological quantities. The redshift can be expressed in terms of the distance angle α [Figure 4.2-1(a)] or the optical distance as

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{D/R_4}{1 - D/R_4} = e^\alpha - 1 \quad ; \quad D = \Delta R_4 = R_4 \frac{z}{1+z} = R_4 \left(e^\alpha - 1 \right) \quad (4.2:1)$$

The optical distance D is the distance light propagates in space directions, tangential directions perpendicular to the 4-radius. Because the velocity of light in space is equal to the velocity the 4-radius increases, D is equal to the increase of the 4-radius ΔR_0 during the propagation time.

In DU space gravitationally bound local systems like galaxies and quasars expand in direct proportion to the expansion of space. Atomic objects conserve their dimensions in expanding space.

The angular size of an *expanding object* with diameter $d = d_R/(1+z)$ at the time light from the object is emitted is
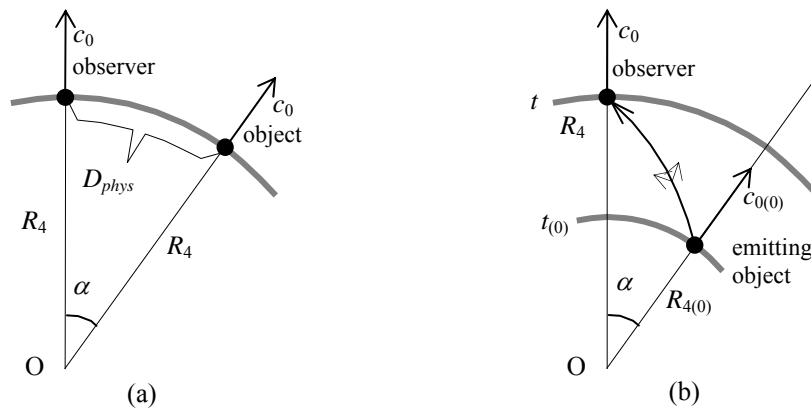

Figure 4.2-1.(a) The classical Hubble law corresponds to Euclidean space where the observed distance of the object is equal to the physical distance, the arc D_{phys}, at the time of the observation. (b) When the propagation time of light from the object is taken into account the observed distance is the optical distance which is the length of the integrated path over which light propagates in the tangential direction on the surface of the expanding 4-sphere. Because the velocity of light in space is equal to the expansion of space in the direction of R_4, the optical distance is $D = R_4 - R_{4(0)}$, the lengthening of the 4-radius during the propagation time.

(a) DU-prediction (Euclidean)   (b) FLRW-prediction

——— $\Omega_m = 1$, $\Omega_\Lambda = 0$ (no dark energy)
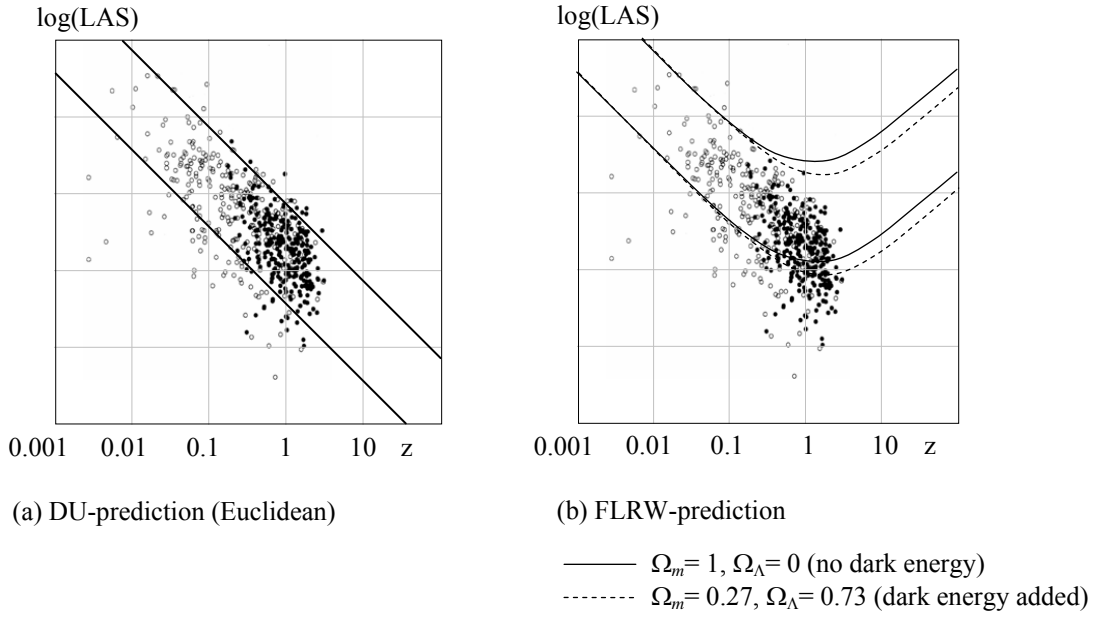-------- $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ (dark energy added)

Figure 4.2-1. Dataset of observed Largest Angular Size (LAS) of quasars and galaxies in the redshift range $0.001 < z < 3$. Open circles are galaxies, filled circles are quasars. (Data collection [11]: K. Nilsson et al., Astrophys. J. 413, 453, 1993). In (a) observations are compared with the DU prediction (4.2:2). In (b) observations are compared with the FLRW prediction with $\Omega_m = 0$ and $\Omega_\Lambda = 0$ (solid curves), and $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$ (dashed curves).

$$\theta = \frac{d}{D} = \frac{d_R}{(1+z)} \frac{(1+z)}{R_4 z} = \frac{d_R}{R_4} \frac{1}{z} = \frac{\alpha_d}{z} \qquad ; \qquad \frac{\theta}{R_4/d_R} = \frac{\theta}{\alpha_d} = \frac{1}{z} \qquad (4.2:2)$$

where the ratio $d_R/R_4 = \alpha_d$ means the angular size of the expanding object as seen from the center of the 4-sphere. Equation (4.2:2) implies Euclidean appearance of expanding objects corresponding to observations of the angular diameter of galaxies and quasars. Equation (4.2:2) implies a major difference from the corresponding prediction of the standard cosmology model, which predicts an increasing angular size at redshifts $z > 2$ (Figure 4.2-1).

As shown in Figure 4.2-1 the DU predictions for angular sizes of galaxies and for magnitudes of supernovas are in excellent agreement with observations without assumptions of dark energy or any additional parameters. The mathematical structure of the predictions is essentially simpler than the mathematical structure in the corresponding predictions of the standard cosmology model.

Conservation of the energy balance in DU space requires conservation of the mass equivalence of radiation propagating in space. The cycle time and wavelength of radiation propagating in expanding space are increased in direct proportion to the expansion of the 4-radius. The DU expression for K-corrected distance modulus in multi-bandpass detection becomes [see Appendix 1]

$$\mu = m - M = 5\log\frac{R_4}{10 \text{ pc}} + 5\log z + 2.5\log(1+z) \qquad (4.2:3)$$

where m is the apparent magnitude and M is the absolute magnitude of the reference source at distance 10 parsec. R_4 in (4.2:3) is the present value of the 4-radius which is about 14 billion light years corresponding to Hubble constant $H_0 \approx 70$ [(km/s)/Mpc]. Equation (4.2:3) is the DU-replacement of the FLRW equation [12]

$$\mu = m - M = 5\log\frac{R_H}{10 \text{ pc}} + 5\log\left[(1+z)\int_0^z \frac{1}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\Lambda}} dz\right] \qquad (4.2:4)$$
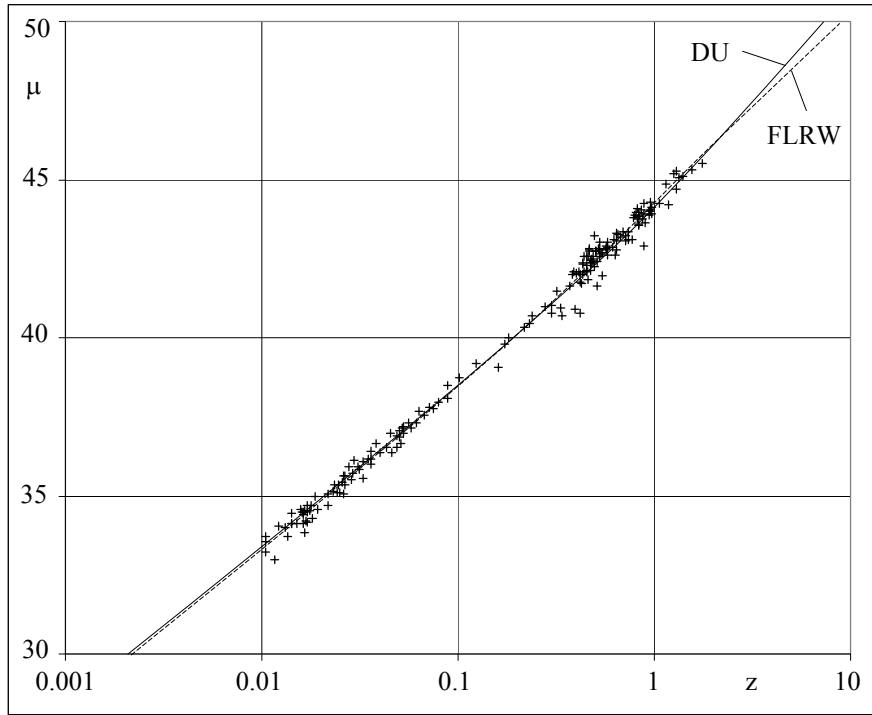
Figure 4.2-2. Distance modulus $\mu = m - M$, vs. redshift for Riess et al. "high-confidence" dataset and the data from the HST, Riess [13]. The optimum fit for the FLRW prediction (4.2:3) is based on $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$. The difference between the DU prediction (4.2:3) and the prediction of the standard model (4.2:4) is very small in the red-shift range covered by observations, but becomes meaningful at redshifts above $z > 3$.

Figure 4.2-2 shows a comparison of the predictions in (4.2:3) and (4.2:4) with Supernova Ia observations [13,14]. The difference between the two predictions is negligible in the redshift range $0 < z < 2$ but becomes meaningful at higher redshifts.

5. Summary

As a holistic approach to physical reality the Dynamic Universe starts from the whole and devolves down to local. It is primarily an analysis of the energy resources available for the manifestation of physical processes and structures in space. The Dynamic Universe relies on absolute time and universal measures of distance as the coordinate quantities essential for human conception. Mass in the Dynamic Universe obtains the meaning of wave-like substance for all expressions of energy and matter.

Relativity in Dynamic Universe appears as a consequence of the finiteness of total energy in space. Relativity is not described in terms of modified coordinate quantities as in the relativity theory but as the locally available rest energy determined by the velocity and gravitational state of the object in the local energy frame and in the system of nested energy frames in space.

Predictions for local observations in DU space are essentially the same as those obtained in the special and general theories of relativity; differences become meaningful at extremes – in the vicinity of local singularities and at cosmological distances.

Instead of a sudden appearance of mass and energy in a Big-bang, the buildup and release of total energy in DU space is a continuous process from infinity in the past to infinity in the future. A "Big-bang-like" singularity of DU space is seen as the turning point of the contraction phase into the ongoing expansion phase. At infinity in the future, the energy of motion gained from gravitational energy in the contraction will have been returned back to the gravitational energy of the structure in the expansion. Mass will be no longer observable because the energy of excitation will have vanished. *The cycle of observable physical existence begins in emptiness in the past and ends in emptiness in the future.*

Appendix 1. Derivation of cosmological predictions

A1.1 Optical distance and the Hubble law

As a consequence of the spherical symmetry and the zero-energy balance in space, the velocity of light is determined by the velocity of space in the fourth dimension. The momentum of electromagnetic radiation has the direction of propagation in space. Although the actual path of light is a spiral in four dimensions, the length of the optical path in the direction of the momentum of radiation in space, is the tangential component of the spiral, which is equal to the increase of the 4-radius, the radial component of the path, during the propagation, Fig. A1.1-1

$$D = R_0 - R_{0(0)} \tag{A1.1:1}$$

The differential of optical distance can be expressed in terms of R_0 and the distance angle α as

$$dD = R_0 d\alpha = c_0 dt = dR_0 \tag{A1.1:2}$$

By first solving for the distance angle α

$$\alpha = \int_{R_{0(0)}}^{R_0} \frac{dR_0}{R_0} = \ln \frac{R_0}{R_{0(0)}} = \ln \frac{R_0}{R_0 - D} \tag{A1.1:3}$$

the optical distance D obtains the form

$$D = R_0 \left(1 - e^{-\alpha}\right) \tag{A1.1:4}$$

where R_0 means the value of the 4-radius at the time of the observation.

The observed recession velocity, the velocity at which the optical distance increases, obtains the form

$$v_{rec(optical)} = \frac{dD}{dt} = c_0 \left(1 - e^{-\alpha}\right) = \frac{D}{R_0} c_0 \tag{A1.1:5}$$

As demonstrated by equation (A1.1:5) the maximum value of the observed optical recession velocity never exceeds the velocity of light, c, at the time of the observation, but approaches it asymptotically when distance D approaches the length of 4-radius R_0.

Atoms conserve their dimensions in expanding space. As shown by Balmer's equation, the characteristic emission wavelength is directly proportional to the Bohr radius, which means that also the characteristic emission wavelengths of atoms are unchanged in the course of the expansion of space. The wavelength of radiation propagating in expanding space is assumed to be subject to increase in direct proportion to the expansion space, Fig. A1.1-1(b). Accordingly, redshift, the increase of the wavelength becomes

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{R_0 - R_{0(0)}}{R_{0(0)}} = \frac{D/R_0}{1 - D/R_0} = e^\alpha - 1 \tag{A1.1:6}$$
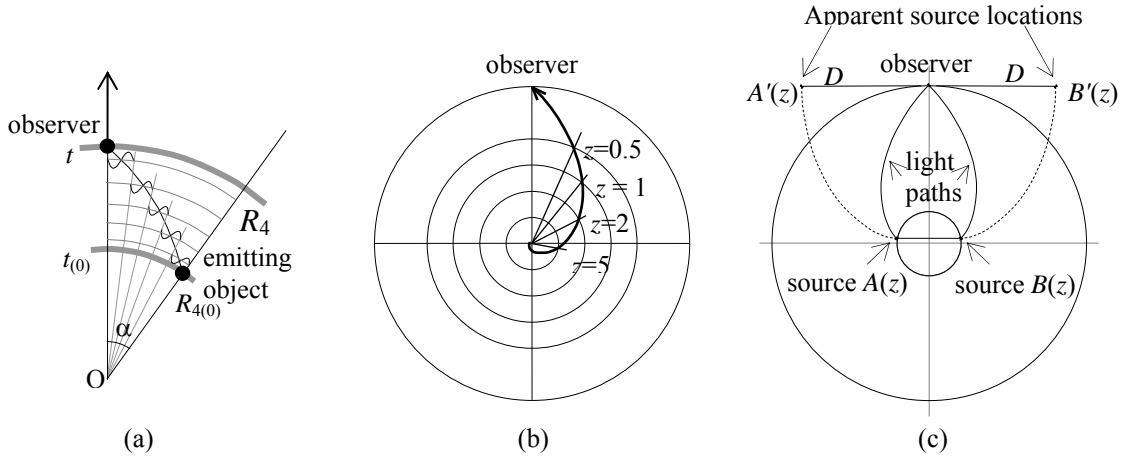
Figure A1.1-1. (a) The redshift of radiation results from the lengthening of the wavelength with the expansion of space. The number of quanta (or wavelengths of radiation) on the way from an emitter of constant intensity to an observer at a fixed distance angle from the emitter is constant with time. (b) Expansion of space during the propagation time of light from objects at different distances: The length of the 4-radius R_4 and the corresponding optical path is indicated for redshifts $z = 0.5$ to 5. (c) Propagation of light in expanding spherically closed space. The apparent line of sight is the straight tangential line. The distance to the apparent source of the light is at the optical distance $D = R_{(observation)} - R_{(emission)}$ along the apparent line of sight. Objects with redshift z, $A(z)$ and $B(z)$, are observed as apparent sources $A'(z)$ and $B'(z)$ on an observer centered 3-dimensional sphere with radius $D = R_0\, z/(1+z\,)$.

where $D = R_0 - R_{0(0)}$ is the optical distance of the object given in (A1.1:4), λ and R_0 are the wavelength and the 4-radius at the time of the observation, respectively, and $R_{0(0)}$ is the 4-radius of space at the time the observed light was emitted, see Fig. A1.1-1(b). Solved from (A1.1-6) the optical distance can be expressed

$$D = R_0 \frac{z}{1+x} = R_0 \left(e^\alpha - 1\right)$$
(A1.1:7)

Space at redshift z is observed as the surface of an observer-centered 3-dimensional sphere with radius D, Fig. A1.1-1(c).

Atoms conserve their dimensions in expanding space. As shown by Balmer's equation, the characteristic emission wavelength is directly proportional to the Bohr radius, which means that also the characteristic emission wavelengths of atoms are unchanged in the course of the expansion of space. The wavelength of radiation propagating in expanding space is assumed to be subject to increase in direct proportion to the expansion space, Fig. A1.1-1(b). Accordingly, redshift, the increase of the wavelength becomes

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{R_0 - R_{0(0)}}{R_{0(0)}} = \frac{D/R_0}{1 - D/R_0} = e^\alpha - 1$$
(A1.1:6)

where $D = R_0 - R_{0(0)}$ is the optical distance of the object given in (A1.1:4), λ and R_0 are the wavelength and the 4-radius at the time of the observation, respectively, and $R_{0(0)}$ is the 4-radius of space at the time the observed light was emitted, see Fig. A1.1-1(b). Solved from (A1.1-6) the optical distance can be expressed

$$D = R_0 \frac{z}{1+x} = R_0 \left(e^\alpha - 1\right)$$
(A1.1:7)

Space at redshift z is observed as the surface of an observer-centered 3-dimensional sphere with radius D, Fig. A1.1-1(c).

The optical distance D of equation (A1.1:7) corresponds closest to the angular diameter distance in the standard model [12]

$$D_A = \frac{R_H}{(1+z)} \int_0^z \frac{1}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\Lambda}} \, dz \qquad (A1.1:8)$$

where the flat space condition, $\Omega_m + \Omega_\Lambda = 1$ is assumed, and $R_H = c/H_0$ is the Hubble radius corresponding to R_0 in DU space. Ω_m and Ω_Λ give the shares of the densities of baryonic plus dark mass and the dark energy in space, respectively. The term "angular diameter distance" refers to the distance converted into the observation angle of a standard rod and non-expanding objects in space.

In FLRW cosmology not only solid objects like stars but also all local systems like galaxies and quasars are non-expanding objects which allows the expression of the observation angle of cosmological objects generally as

$$\theta = \frac{d}{D_A} = \frac{d}{R_H}(1+z) \Big/ \int_0^z \frac{1}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\Lambda}} \, dz \qquad (A1.1:9)$$

The observation angle of a standard rod or *non-expanding objects* (solid objects like stars) in DU space is

$$\theta = \frac{d_{rod}}{D} = \frac{d_{rod}}{R_4}\frac{(1+z)}{z} \qquad ; \qquad \frac{\theta}{d_{rod}/R_4} = \frac{(1+z)}{z} \qquad (A1.1:10)$$

As shown by equation (A1.1:10), the observation angle of a standard rod approaches the size angle $\alpha_d = d_{rod}/R_4$ of the object at high redshift ($z \gg 1$).

In DU space gravitationally bound local systems expand in direct proportion to the expansion of space. The angular size of an *expanding object* with diameter $d = d_R/(1+z)$ at the time light from the object is emitted is

$$\theta = \frac{d}{D} = \frac{d_R}{(1+z)}\frac{(1+z)}{R_4 z} = \frac{d_R}{R_4}\frac{1}{z} = \frac{\alpha_d}{z} \qquad ; \qquad \frac{\theta}{R_4/d_R} = \frac{\theta}{\alpha_d} = \frac{1}{z} \qquad (A1.1:11)$$

where the ratio $d_R/R_4 = \alpha_d$ means the angular size of the expanding object as seen from the center of the 4-sphere. Equation (A1.1:11) implies Euclidean appearance of expanding objects.

The standard model of FLRW space defines two other distance quantities related to the angular diameter distance. The *co-moving distance* is the distance of objects as it is at the time of observation, i.e. excluding the light propagation time from the object. The co-moving distance in the FLRW space is

$$D_{Co-moving} = (1+z)D_A = R_H \int_0^z \frac{1}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\Lambda}} \, dz \qquad (A1.1:12)$$

The DU equivalence of co-moving distance is the physical distance measured along the curved surface of spherically closed space

$$D_{phys} = \alpha R_0 = R_0 \ln(1+z) \qquad (A1.1:13)$$

Luminosity distance in FLRW space is the distance equivalence (in parsec) used to convert distance into *magnitude* using the classical definition of magnitude

$$M = m - 5 \cdot \log(D_L - 1) \qquad ; \qquad D_L = 10^{\frac{(m-M)}{5}+1} \qquad (A1.1:14)$$

or in a more illustrative form to give the apparent magnitude m in equation

Distances in FLRW space:
   a) Luminosity distance, D_L
   b) Co-moving distance, $D_{co\text{-}moving}$
   c) Angular diameter distance, D_A

Distances in DU space:
   a) Luminosity distance, $D_{L(DU)}$
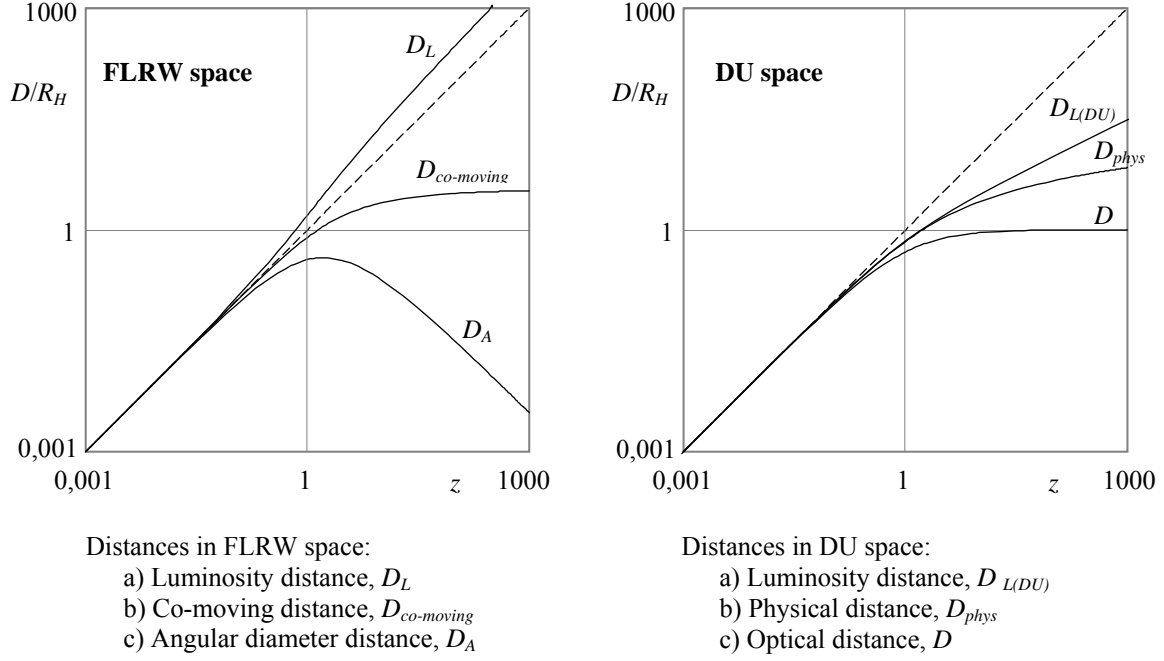   b) Physical distance, D_{phys}
   c) Optical distance, D

Figure A1.1-2. Comparison of distance definitions in FLRW space and DU space. The dashed line in both figures is the linear distance corresponding to classical Hubble law $D = H_0\, z$.

$$m = M + 2.5 \cdot \log\left[\left(\frac{R_H}{d_0}\right)^2\right] + 2.5 \cdot \log\left[\left(\frac{D_L}{R_H}\right)^2\right] \qquad (A1.1\!:\!15)$$

where M is the absolute magnitude of the reference source at distance $d_0 = 10$ pc. The Luminosity distance in FLRW cosmology is

$$D_L = \left(1+z\right)^2 D_A = R_H \left(1+z\right)\int_0^z \frac{1}{\sqrt{\left(1+z\right)^2\left(1+\Omega_m z\right) - z\left(2+z\right)\Omega_\Lambda}}\, dz \qquad (A1.1\!:\!16)$$

which assumes factor $(1+z)^2$ for the redshift dilution in the observed power density (see section A1.2) and another $(1+z)^2$ "aberration factor" for the spreading of radiation due to expansion. The magnitude prediction based on luminosity distance D_L in FLRW cosmology assumes reduction of the observed power densities to power densities in "emitter's rest frame" by a $(1+z)$ factor in the *K-correction* which is classically used as the instrumental correction for redshifted spectrum, see sections A1.3 and A1.4.

In DU space the dilution factor of redshift is $(1+z)$. In DU space, luminosity distance for observed bolometric power density is

$$D_{L(DU)} = D\sqrt{1+z} = R_0 \cdot \frac{z}{1+z}\sqrt{1+z} = R_0 \cdot z\sqrt{1+z} \qquad (A1.1\!:\!18)$$

The physical basis of the redshift dilution is discussed in section A1.2. Figure A1.1-2 compares the distance definitions in FLRW space and DU space.

A1.2 The effects of redshift and distance on electromagnetic radiation

In the DU framework the Coulomb energy and the energy of electromagnetic radiation can be expressed in terms of a mass equivalence and the velocity of light, formally, like the rest energy of matter.

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Matter: $$E_{rest} = mc_0c \qquad \text{(A1.2:1)}$$

Cycle (N^2 quanta) of electromagnetic radiation: $$E_\lambda = N^2 \frac{h_0}{\lambda_e} c_0 c = m_\lambda c_0 c \qquad \text{(A1.2:2)}$$

Coulomb energy: $$E_C = N_1 N_2 \alpha \frac{h_0}{2\pi r} c_0 c = m_C c_0 c \qquad \text{(A1.2:3)}$$

Conserving the mass equivalence of a quantum of radiation, the energy flux of electromagnetic radiation becomes

$$F_{rec} = E_\lambda f = \frac{h_0}{\lambda_e} c c_0 \cdot f = \frac{h_0}{\lambda_e} \frac{c_0 c^2}{\lambda_e (1+z)} = \frac{h_0 c_0 c^2}{\lambda_e^2 (1+z)} \qquad \text{(A1.2:4)}$$

where λ_e is the wavelength of radiation at the emission. The reference flux emitted by an identical source at the time and location the redshifted radiation is received ($\lambda_r = \lambda_e$) is

$$F_{emit(ref)} = E_\lambda f = \frac{h_0}{\lambda_e} c c_0 f = m_{\lambda_e} c c_0 \frac{c}{\lambda_e} = \frac{h_0}{\lambda_e} \frac{c_0 c^2}{\lambda_e} = \frac{h_0 c_0 c^2}{\lambda_e^2} \qquad \text{(A1.2:5)}$$

Relative to the reference flux, the power density in the redshifted flux is

$$F_{rec} = \frac{F_{emit(ref)}}{(1+z)} \qquad \text{(A1.2:6)}$$

In DU space, the energy flux observed in radiation redshifted by z is diluted by factor $(1+z)$, not by factor $(1+z)^2$ as assumed in the standard model solution [15]. The difference comes from the interpretation of the effect of redshift on the energy of a quantum. As first proposed by Hubble and Humason [16] and later by de Sitter [17], the energy of a quantum is reduced by $(1+z)$ as a consequence of the effect of Planck's equation $E = hf$ as a reduction of the "intensity of the radiation". When receiving the redshifted radiation at a lowered frequency, a second $(1+z)$ factor was assumed. Hubble [18] considered that the latter is relevant only in the case that the redshift is due to recession velocity [19]. The first $(1+z)$ factor was called the "energy effect" and the second $(1+z)$ factor the "number effect".

Conservation of the mass equivalence of radiation in DU space negates the basis for an "energy effect" as a violation of the conservation of energy. An analysis of the linkage between Planck's equation and Maxwell's equations shows that Planck's equation describes the energy conversion at the *emission* of electromagnetic radiation. Redshift should be understood as *dilution of the energy density due to an increase in the wavelength* in the direction of propagation, not as *losing of energy*. Accordingly, the observed energy flux $F = E_\lambda f$ is subject only to a single $(1+z)$ dilution factor, the "number effect" in the historical terms.

Referring to equation (A1.2:4), at distance D from source A the density of the energy flux F_A is

$$F_A = \frac{N^2}{4\pi D^2} \frac{h_0 c_0 c^2}{\lambda_e^2 (1+z)} \qquad \text{(A1.2:7)}$$

where N is the intensity factor of the source. Related to the flux density F_B from a reference source B with same intensity at distance d_0 ($z \approx 0$) the energy flux F_A is

$$F_A = F_B \cdot \frac{N^2}{4\pi D^2} \frac{h_0 c_0 c^2}{\lambda_e^2 (1+z)} \bigg/ \frac{N^2}{4\pi d_0^2} \frac{h_0 c_0 c^2}{\lambda_e^2} = F_B \cdot \frac{d_0^2}{D^2} \frac{1}{(1+z)} \qquad \text{(A1.2:8)}$$

Substitution of equation (A1.1:7) for D in (A1.2:8) gives

$$F_A = F_B \cdot \frac{d_0^2}{R_4^2} \frac{(1+z)^2}{z^2} \frac{1}{(1+z)} = F_B \cdot \frac{d_0^2}{R_4^2} \frac{(1+z)}{z^2} \qquad (A1.2:9)$$

For a $d_0 = 10$ pc reference source, $F_B = F_{10pc}$ we get the expression for the apparent magnitude

$$m = M - 2.5 \log \frac{F_A}{F_{10pc}} = M + 5 \log \frac{R_4}{d_0} + 5 \log z - 2.5 \log(1+z) \qquad (A1.2:10)$$

where M is the absolute magnitude of the reference source at distance d_0.

Equation (A1.2:10) applies for the *bolometric energy flux* observed for radiation from a source at optical distance (angular size distance) $D = R_4 \cdot z / (1+z)$ from the observer in DU space. Equation (A1.2:10) does not include possible effects of galactic extinction, spectral distortion in Earth atmosphere, or effects due to the local motion and gravitational environment of the source and the receiver.

In the present practice, apparent magnitudes are expressed as *K*-corrected magnitudes which in addition to instrumental factors for bolometric magnitude include a "correction to source rest frame" required by the prediction of the apparent magnitude in the standard cosmology model. To make the DU prediction in equation (A1.2:10) consistent with the *K*-corrected magnitudes assumed in the FLRW prediction, equation (A1.2:10) is be complemented as

$$m_{(K)} = M + 5 \log \frac{R_4}{D_0} + 5 \log z - 2.5 \log(1+z) + K \qquad (A1.2:11)$$

The *K*-correction is discussed in detail in section A1.4.

A1.3 Multi-bandpass detection

For analyzing the detection of bolometric flux densities and magnitudes by multi-bandpass photometry the source radiation is assumed to have the spectrum of blackbody radiation. The bandpass system applied consists of a set of UBVIZYJHK filters approximated with transmissions curves of the form of normal distribution

$$f_X(\lambda) = e^{-\left[(\lambda - \lambda_{0X})^2 / 2(\Delta \lambda_X / \sigma_{\frac{1}{2}})^2\right]} = e^{-\frac{\sigma_{\frac{1}{2}}^2}{2} \frac{\lambda_{C(X)}}{\Delta \lambda_X} \left(\frac{\lambda}{\lambda_{C(X)}} - 1\right)^2} = e^{-\frac{2.773}{W_X^2} \left(\frac{\lambda}{\lambda_{C(X)}} - 1\right)^2} \qquad (A1.3:1)$$

where $\lambda_{C(X)}$ is the peak wavelength of filter X, $\Delta \lambda_X$ the half width of the filter, $W_X = \Delta \lambda_X / \lambda_{C(X)}$ the relative width, and $\sigma_{\frac{1}{2}} = 2.35481$ is half width deviation of the normal distribution (Fig. A1.3-1).

For the numerical calculation of the energy flux from a blackbody source, equation (A.2:10) in Appendix A2 is rewritten for a relative wavelength-differential $d\lambda_z / \lambda_z = d\lambda/\lambda << W_X$

$$F\left(\frac{d\lambda_z}{\lambda_z}\right) = \frac{15}{\pi^4} \frac{F_{bol(z=0)}}{(1+z)} \left[\left(\lambda_0 \middle/ \frac{\lambda}{1+z}\right)^4 \middle/ \left(e^{\left(\lambda_0 \middle/ \frac{\lambda}{1+z}\right)} - 1\right)\right] \frac{d\lambda}{\lambda} \qquad (A1.3:2)$$

Equation (A1.3:2) excludes the dilution due to the distance from the source to the observer. Integration of (A1.3:2) gives the bolometric radiation

$$F_{bol} = \int_0^\infty F\left(\frac{d\lambda_z}{\lambda_z}\right) = \frac{F_{bol(z=0)}}{1+z} \qquad (A1.3:3)$$

The transmission through filter X, normalized to the bolometric flux by applying equation (A2:12), can now be calculated by applying the transmission function of equation (A1.3:1) to the flux in (A1.3:2)
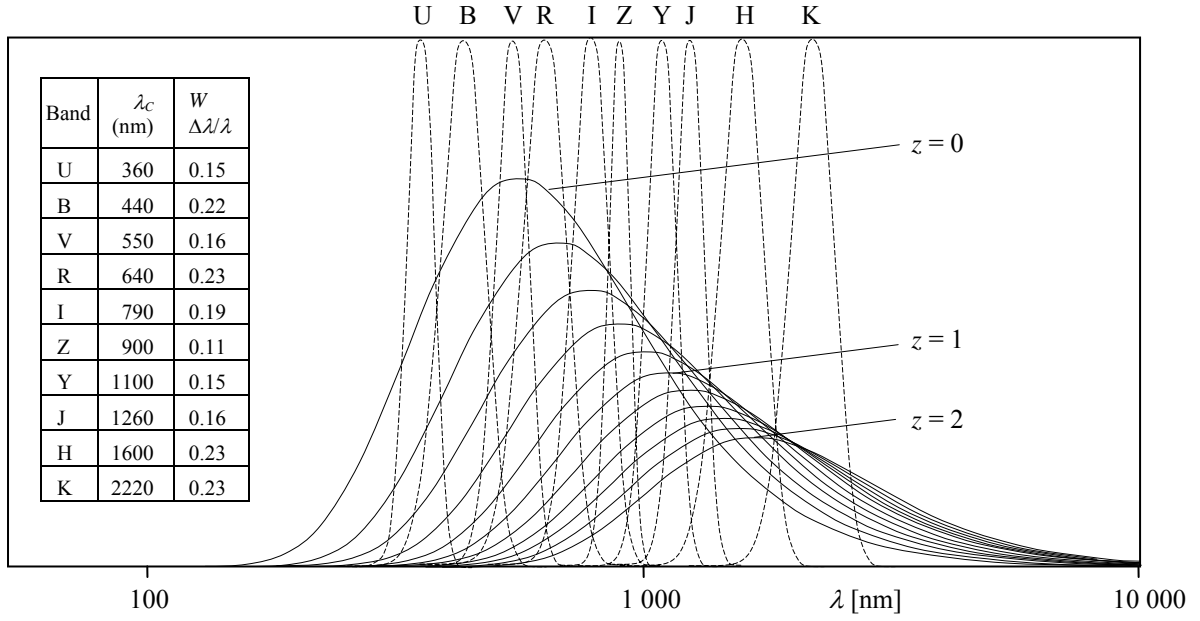
U B V R   I Z Y J   H   K

Band	λ_C (nm)	W $\Delta\lambda/\lambda$
U	360	0.15
B	440	0.22
V	550	0.16
R	640	0.23
I	790	0.19
Z	900	0.11
Y	1100	0.15
J	1260	0.16
H	1600	0.23
K	2220	0.23

Figure A1.3-1. The effect of redshift $z = 0\ldots2$ (shown in steps of 0.2) on the energy flux density per relative bandwidth of the blackbody radiation spectrum from a $T = 6600$ °K blackbody source corresponding to $\lambda_T = 440$ nm and $\lambda_W = 557$ nm (solid curves). Transmission curves of UBVRIZYJHK filters listed in the table are shown with dashed lines. The half widths of the filters follow the widths of standard filters in the Johnson system. All transmission curves are approximated with a normal distribution. The horizontal axis shows the wavelength in nanometers in a logarithmic scale.

$$dF_{X(z)}\left(\frac{d\lambda_z}{\lambda_z}\right) = \frac{1}{4.780}\frac{F_{bol(z=0)}}{(1+z)}\int_0^\infty \left[\left(\lambda_0 \Big/ \frac{\lambda}{1+z}\right)^4 \Big/ \left(e^{\left(\lambda_0 / \frac{\lambda}{1+z}\right)}-1\right)\right] e^{-\frac{2.773}{W_X^2}\left(\frac{\lambda/(1+z)}{\lambda_{C(X)}}-1\right)^2} \frac{d\lambda}{\lambda} \quad (A1.3:4)$$

which gives the flux observed through filter X as a function of the redshift of the radiation. Figure A1.3-2 shows the normalized transmission curves calculated for filters UBVRIZJ by integration of (A1.3:4). Each curve touches the bolometric curve (A1.3:3) at the redshift matching the maximum of the radiation flux to the nominal wavelength of the filter.

The energy flux of equation (A1.3:4) from sources at a small distance d_0 ($z_{d0} \approx 0$) and at distance D ($z_D > 0$) are related

$$\frac{F_{X(D)}}{F_{X0(d_0)}} = \frac{d_0^2}{D^2}\frac{\int_0^\infty dF_{X(z)}}{\int_0^\infty dF_{X0(0)}} \quad (A1.3:5)$$

Substitution of equation (A1.1:7) for D and equation (A1.3:4) for $F_{X(D)}$ and $F_{X0(do)}$ in (A1.3:5) gives the radiation power observed in filters X and $X0$ from standard sources at distances D and d_0, respectively

$$\frac{F_{X1(D)}}{F_{X2(d_0)}} = \frac{d_0^2}{R_4^2}\frac{(1+z)^2}{z^2}\frac{1}{(1+z)}\frac{\int_0^\infty \left[\left(\lambda_0 \Big/ \frac{\lambda}{1+z}\right)^4 \Big/ \left(e^{\left(\lambda_0 / \frac{\lambda}{1+z}\right)}-1\right)\right] e^{-\frac{2.773}{W_{X1}^2}\left(\frac{\lambda/1+z}{\lambda_{C(X1)}}-1\right)^2}\frac{d\lambda}{\lambda}}{\int_0^\infty \left[\left(\lambda_0/\lambda\right)^4 \Big/ \left(e^{\left(\lambda_0/\lambda\right)}-1\right)\right] e^{-\frac{2.773}{W_{X2}^2}\left(\frac{\lambda}{\lambda_{C(X2)}}-1\right)^2}\frac{d\lambda}{\lambda}} \quad (A1.3:6)$$
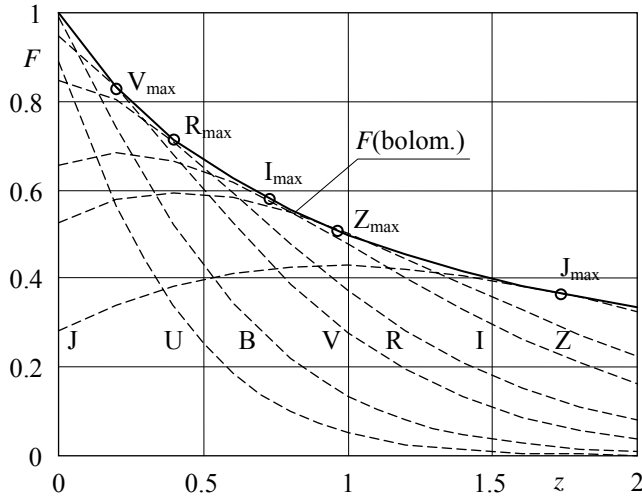
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Figure A1.3-2. Transmission curves obtained by numerical integration of (58) for filters UBVRIZJ for radiation in the redshift range $z = 0\ldots2$ from a blackbody with $\lambda_T = 350$ nm ($\lambda_W = 440$ nm, $T = 8300$ °K). Each curve touches the bolometric curve of equation (57) at the redshift matching maximum of the radiation flux to the nominal wavelength λ_W of the filter (small circles in the figure).

By denoting the integrals in the numerator and denominator in (A1.3:6) by $I_{X(D)}$ and $I_{X0(do)}$, respectively, energy flux $F_{X(D)}$ can be expressed

$$F_{X(D)} = F_{X0(d_0)} \frac{d_0^2}{R_4^2} \frac{(1+z)}{z^2} \frac{I_{X(D)}}{I_{X0(d_0)}} \tag{A1.3:7}$$

Choosing $d_0 = 10$ pc, the apparent magnitude for flux through filter X at distance D can be expressed as

$$m_{X1} = M + 5\log\left(\frac{R_4}{10\text{pc}}\right) + 5\log(z) - 2.5\log(1+z) + 2.5\log\left(\frac{I_{X2(d_0)}}{I_{X1(D)}}\right) \tag{A1.3:8}$$

where M is the absolute magnitude of the reference source at distance 10 pc.

For $R_4 = 14 \cdot 10^9$ l.y., consistent with Hubble constant $H_0 = 70$ [(km/s)/Mpc], the numerical value of the second term in (A1.3:8) is $5 \cdot \log(R_4/10\text{pc}) = 43.16$ magnitude units. For Ia supernovae the numerical value for the absolute magnitude is about $M \approx 19.5$.

When filter X is chosen to match $\lambda_{C(X)} = \lambda_W(1+z)$ and $\lambda_{C(X0)} = \lambda_W$ [or $\lambda_{C(X)} = \lambda_T(1+z)$ and $\lambda_{C(X0)} = \lambda_T$], the integrals $I_{X(D)}$ and $I_{X0(do)}$ are related as the relative bandwidths

$$\frac{I_{X0(d_0)}}{I_{X(D)}} = \frac{W_{X0}}{W_X} \tag{A1.3:9}$$

which means that for optimally chosen filters with equal relative widths the last term in equation (A1.3:8) is zero and equation (A1.3:8) obtains the form of equation (A1.2:10) for bolometric energy flux

$$m_{X(opt)} = M + 5\log\left(\frac{R_4}{10\text{pc}}\right) + 5\log(z) - 2.5\log(1+z) \tag{A1.3:10}$$

Figure A1.3-3 illustrates magnitudes calculated for filters $X = B, V, R, I, Z, J$ from equation (A1.3:8) in the redshift range $z = 0\ldots2$. Each curve touches the solid curve of equation (A1.3:10) corresponding to the bolometric magnitude obtainable with optimal filters at each redshift in the redshift range studied. The predictions are compared to observed magnitudes, Tonry et al. [20], Fig. A1.3-3(c).
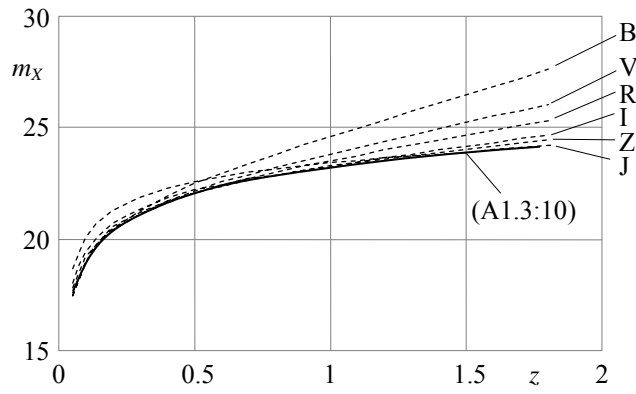
(a) $\lambda_T = 350$ nm ($\lambda_W = 440$ nm, $T = 8300$ °K)
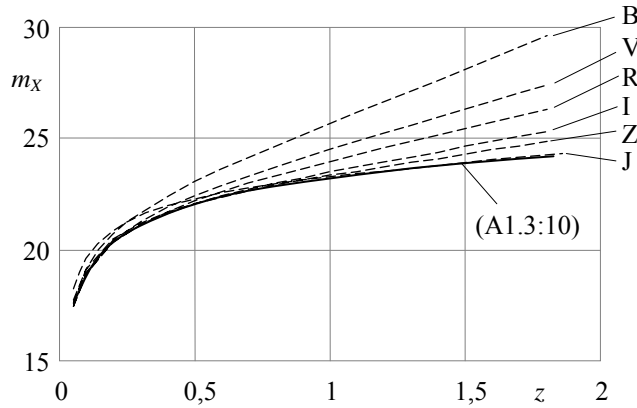
(b) $\lambda_T = 440$ nm ($\lambda_W = 560$ nm, $T = 6600$ °K)
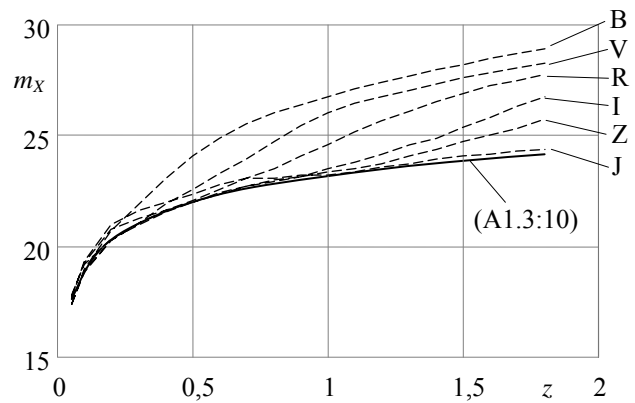
(c) Observed magnitudes Tonry et al., [34], Table 14.

Figure A1.3-3 (a, b) Predicted magnitudes (A1.3:8) for filters BVRIZJ as functions of redshift are shown as the families of curves drawn with dashed line. The blackbody temperature in (a) is 8300 °K and 6600 °K in (b), see Appendix A2 for the definitions of λ_T and λ_W characterizing blackbody radiation. (c) Plot of the peak magnitudes of normal Sn Ia observed in BVRIYJ filters as presented by Tonry et al. [22] in Table 14. The transmission functions of the filters used by Tonry et al. are slightly different from the transmission functions used in calculations for (a) and (b). The DU prediction (A1.3:10) for the magnitudes in optimally chosen filters is shown by the solid DU curve in each figure.

A1.4 K-corrected magnitudes

In the observation praxis based on Standard Cosmology Model, direct observations of magnitudes in the bandpass filters are treated with *K-correction* which corrects the filter mismatch and converts the observed magnitude to the "emitter's rest frame" presented by observations in a bandpass matched to a low redshift reference of the objects studied. The *K-correction* for observations in the X_j band relative to the rest frame reference in the X_i band is defined [21]

$$K_{i,j}(z) = 2.5\log(1+z) + 2.5\log\left\{\frac{\displaystyle\int_0^\infty F(\lambda)S_i(\lambda)\,d\lambda \quad \int_0^\infty Z(\lambda)S_j(\lambda)\,d\lambda}{\displaystyle\int_0^\infty F(\lambda/(1+z))S_j\,d\lambda \quad \int_0^\infty Z(\lambda)S_i(\lambda)\,d\lambda}\right\} \qquad (A1.4{:}1)$$

In the case of a blackbody source and filters with transmission functions described by a normal distribution, equation (A1.4:1) can be expressed by substituting equation (A1.3:2) for the energy flux integrals, equation (A1.3:1) for the transmission curves of the filters, and the relative bandwidths of filters i and j for the transmission integrals

$$K_{i,j(W)}(z) = 2.5\log(1+z)$$

$$+2.5\log\left\{\frac{\displaystyle\int_0^\infty \left[(\lambda_0/\lambda)^5 \big/ (e^{(\lambda_0/\lambda)}-1)\right] e^{-\frac{2.773}{W_i^2}\left(\frac{\lambda}{\lambda_{C(i)}}-1\right)^2} d\lambda}{\dfrac{1}{1+z}\displaystyle\int_0^\infty \left[\left(\lambda_0 \Big/ \frac{\lambda}{1+z}\right)^5 \Big/ \left(e^{\left(\lambda_0 \big/ \frac{\lambda}{1+z}\right)}-1\right)\right] e^{-\frac{2.773}{W_j^2}\left(\frac{\lambda/(1+z)}{\lambda_{C(j)}}-1\right)^2} d\lambda} \cdot \frac{W_j}{W_i}\right\} \qquad (A1.4{:}2)$$

where the relative differential $d\lambda/\lambda$ of (A1.3:2) is replaced by differential $d\lambda$ to meet the definition of (A1.4:1). Figure A1.4-1 (a) illustrates the K_{BX}-corrections calculated for radiation from a blackbody source with $\lambda_T = 440$ nm equivalent to 6600 °K blackbody temperature. An optimal choice of filters, matching the central wavelength of the filter to the wavelength of the maximum of redshifted radiation, leads to the K-correction

$$K(z) \approx 5\log(1+z) \qquad (A1.4{:}3)$$

with an accuracy of better than 0.1 magnitude units in the whole range of redshifts covered with the set filters used. The difference between the K-corrections in equation (A1.4:2) and (A1.4:3) is presented in Figure A1.4-1(b).

Substitution of (A1.4:3) for K in equation (A1.2:11) gives the DU space prediction for K-corrected magnitudes
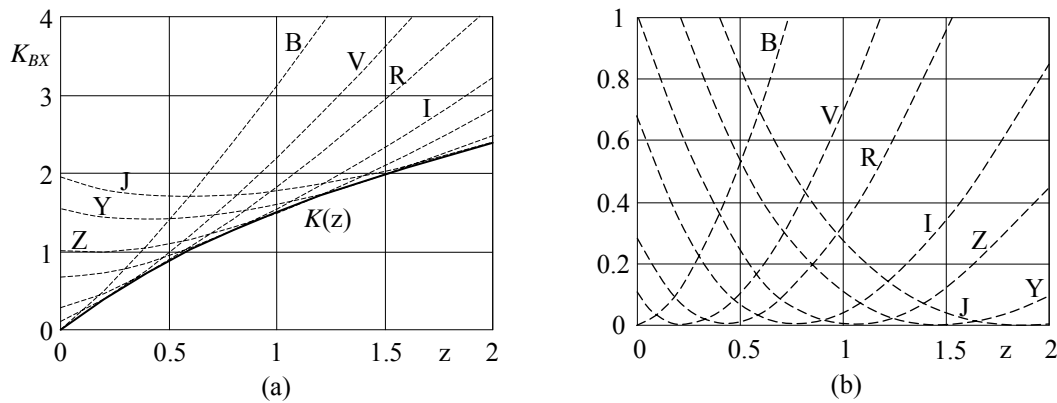

Figure A1.4-1. (a) K_{BX}-corrections (in magnitude units) according to (A1.4:2) for B band as the reference frame, calculated in the redshift range $z = 0\ldots2$ for radiation from a blackbody source with $\lambda_T = 440$ nm equivalent to 6600 °K blackbody temperature. All K_{BX}-correction curves touch the solid $K(z)$ curve, which shows the $K(z) = 5\cdot\log(1+z)$ function. (b) The difference $K_{BX} - K(z)$. With an optimal choice of filters, the difference $K_{BX} - K(z)$ is smaller than 0.05 magnitude units in the whole range of redshifts $z = 0\ldots2$ covered by the set of filters B…J demonstrating the bolometric detection with optimally chosen filters.
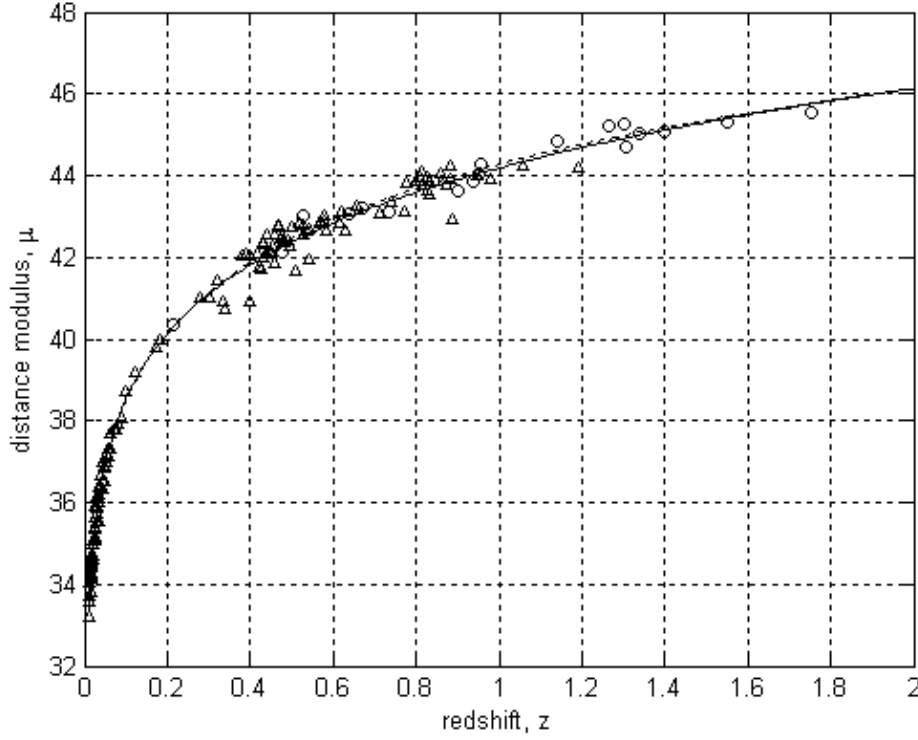
Figure A1.4-2. Distance modulus $\mu = m - M$, vs. redshift for Riess et al.'s gold dataset and the data from the HST. The triangles represent data obtained via ground-based observations, and the circles represent data obtained by the HST [23]. The optimum fit for the standard cosmology prediction (A1.4:5) is shown by the dashed curve, and the fit for the DU prediction (A1.4:4) is shown, slightly below, by the solid curve [14].

$$m_{X(opt)} = M + 5\log\frac{R_4}{D_0} + 5\log z + 2.5\log(1+z) \qquad (A1.4:4)$$

The prediction for K-corrected magnitudes in the standard model is given by equation

$$
\begin{aligned}
m &= M + 5\log\left(\frac{R_H}{10\ \text{pc}}\right) + 5\log\left(\frac{D_L}{R_H}\right) \\
&= M + 43.2 + 5\log\left[(1+z)\int_0^z \frac{1}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\lambda}}\, dz\right]
\end{aligned}
\qquad (A1.4:5)
$$

where $R_H = c/H_0 \approx 14\cdot10^9$ l.y. is the Hubble distance, the standard model replacement of R_4 in DU space, and D_L the luminosity distance defined in equation (A1.1:16). Mass density parameters Ω_m and Ω_Λ give the density shares of mass and dark energy in space. For a flat space condition the sum $\Omega_m + \Omega_\Lambda = 1$.

The best fit of equation (A1.4:5) to the K-corrected magnitudes of Ia supernova observations has been obtained with $\Omega_m = 0.26 \ldots 0.31$ and $\Omega_\Lambda = 0.74\ldots0.69$ [24\ldots32]. Figure A1.4-2 shows a comparison of the prediction given by equation (A1.4:5) with $\Omega_m \approx 0.31$, $\Omega_\lambda \approx 0.69\ \Omega$ and $H_0 = 64.3$ used by Riess et al. [25] and the DU space prediction for K-corrected magnitudes in equation (A1.4:4).

In the redshift range $z = 0\ldots2$ the apparent magnitude of equation (A1.4:5) coincides accurately with the magnitudes of equation (A1.4:4). The K-corrections used by Riess et al. [24], Table 2, follow the $K(z) = 5\cdot\log(1+z)$ prediction of equation (A1.4:3), Fig. A1.4-3.
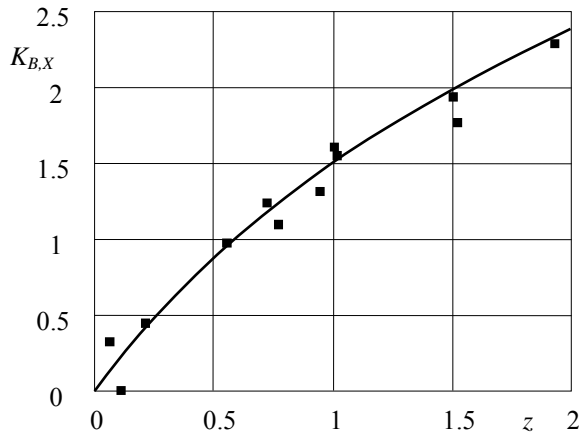
31

Figure A14-3. Average $K_{B,X}$-corrections (black squares) collected from the $K_{B,X}$ data in Table 2 used by Riess et al. (2004) for the K-corrected distance modulus data shown in Figure A1.4-2. The solid curve gives the theoretical K-correction (A1.4:3), $K = 5 \cdot \log(1+z)$, derived for filters matched to redshifted spectra (see Fig. A1.4-1) and applied in equation (A1.4:4) for the DU prediction for K corrected apparent magnitude.

At redshifts above $z > 2$ the difference between the two predictions, (A1.4:4) and (A1.4:5), becomes noticeable and grows up to several magnitude units at $z > 10$, Fig. A1.4-4. For comparison, Figure A1.4-4 shows also the standard model prediction for $\Omega_m = 1$ and $\Omega_\lambda = 0$.
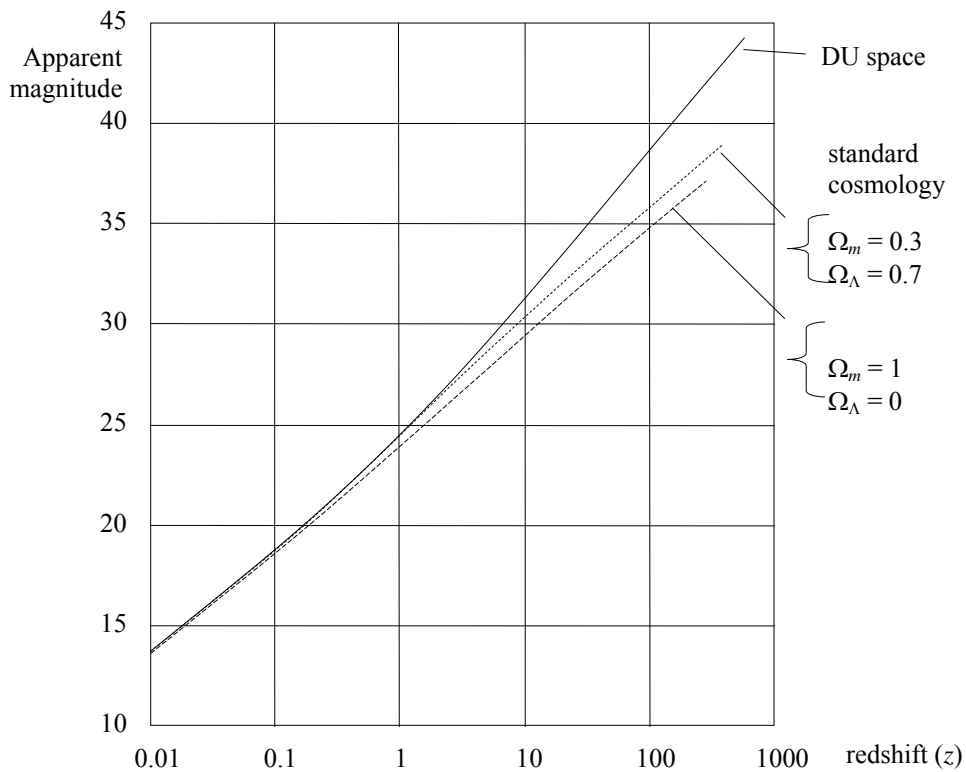

Figure A1.4-4. Comparison of predictions for the K-corrected apparent magnitude of standard sources in the redshift range 0.01...1000 given by the Standard Cosmology Model with $\Omega_m=0.3/\Omega_\Lambda=0.7$ and $\Omega_m=1/\Omega_\Lambda=0$ according to equation (A1.4:5), and DU space given by equation (A1.4:4). In each curve the absolute magnitude used is $M = -19.5$. The $\Omega_m=0.3/\Omega_\Lambda=0.7$ prediction follows the DU prediction closely up to redshift $z \approx 2$, the $\Omega_m=1/\Omega_\Lambda=0$ prediction of the standard model shows remarkable deviation even at smaller redshifts.

A1.5 Surface brightness of expanding and non-expanding objects

The Tolman test [19], [32], [33], and [34] is considered as a critical test for an expanding universe model. In expanding space, according to Tolman's prediction, the observed surface brightness of standard objects decreases by the factor $(1+z)^4$ with the redshift. Two of the four $(1+z)$ factors are explained as consequences of the redshift on the radiation received: a decrease in the arrival rate (the number effect) and in the energy of photons (the energy effect), as discussed in Section A1.2. The two additional $(1+z)$ factors are explained as an apparent increase in the observed area due to aberration.

With reference to equation (A1.1:11) the angular area of an expanding object like a galaxy with a present radius r_e is

$$\Omega_D = \left(\frac{r_{e(z)}}{D}\right)^2 = \frac{r_e^2}{(1+z)^2}\frac{(1+z)^2}{R_4^2 z^2} = \frac{r_e^2}{R_4^2}\frac{1}{z^2} \tag{A1.5:1}$$

where D is the optical distance of the object. Accordingly, the observed bolometric surface brightness of the object is obtained by dividing the bolometric energy flux of equation (A1.2:9) by the angular size of equation (A1.5:1)

$$SB_{(D)} = \frac{F_D}{\Omega_D} = \frac{N^2}{2\pi}\frac{h_0 c_0 c^2}{\lambda_e^2 (1+z)}\frac{(1+z)^2}{r_e^2} = \frac{N^2}{2\pi r_e^2}\frac{h_0 c_0 c^2 (1+z)}{\lambda_e^2} \tag{A1.5:2}$$

Compared to the surface brightness $SB_{(do)}$ of a reference object at distance d_0 with $z_{do} \ll 1$, the observed bolometric surface brightness $SB_{(D)}$ is

$$SB_{(D)} = SB_{(d_0)}\left(\frac{N^2}{2\pi r_e^2}\frac{h_0 c_0 c^2 (1+z)}{\lambda_e^2}\right) \bigg/ \left(\frac{N^2}{2\pi r_e^2}\frac{h_0 c_0 c^2}{\lambda_e^2}\right) = SB_{(d_0)}(1+z) \tag{A1.5:3}$$

or related to the *K*-corrected energy fluxes in multi-bandpass system with nominal filter wavelengths matched to the redshifted radiation [see Section A1.4] as

$$SB_{K(D)} = SB_{(d_0)}(1+z)^{-1} \tag{A1.5:4}$$

The predictions of equations (A1.5:3) and (A1.5:4) do not include the effects of possible evolutionary factors.

In [35–38] Lubin and Sandage give a thorough review of the theoretical and observational aspects of the Tolman $(1+z)^{-4}$ surface brightness prediction as a test of the FLRW expansion. They conclude that observations of the light curves from supernovas have confirmed the cosmological time dilation [39] as a unique proof of an expanding space. They also interpret the precise Planckian shape of the background radiation as a solid proof of the Tolman surface brightness prediction. However, the observed surface brightnesses of high z objects do not follow the Tolman $(1+z)^{-4}$ prediction without assumptions of remarkable evolution in the luminosity and size of the objects.

Galaxy surface brightness and size analysis [40] of HST WFPC2 data in the redshift range $z = 0\ldots4$ shows a qualitative fit of observed surface brightnesses to equation (A1.5:4). Also, the observed reduction in the half-light radius with an increasing redshift is in line with the Euclidean appearance of galaxy space in the DU framework. A detailed analysis of the fit of surface brightness observations to predictions (A1.5:3) and (A1.5:4) is left outside the scope of this paper.

A1.6 The effects of the declining velocity of light

As a consequence of the conservation of the zero-energy condition assumed, all velocities in space are related to the velocity of light determined by the expansion in the direction of the 4-radius. Emission of quanta from a supernova explosion occurs at a frequency proportional to the velocity of light at the time of the explosion. A sequence of waves from an explosion is redshifted

and accordingly received lengthened in the same ratio as the wavelengths are lengthened, i.e. in direct proportion to (1+z). In the standard model, the lengthening is referred to as cosmological time dilation, in DU space it is a direct consequence of reduced velocity of light at the time the wave sequence is received.

The declining rest energy of matter in DU space makes all atomic processes slow down with the expansion of space; ticking frequencies of atomic clocks and the rate of nuclear decay slow down in direct proportion to the decrease of the velocity of light. The present estimates for the oldest globular clusters, based on constant decay rates observed today, are in the range of 12 to 20 billion years [12].

The age of expanding DU space is $T = (2/3) \cdot R_4/c = (2/3)/H_0$ which means about 9.3 billion years for $R_4 = 14$ billion light years consistent with Hubble constant $H_0 = 70$ [(km/s)/Mpc]. Linear age estimates up to 14 billion years are reduced below the age of 9.3 billion years, Fig. A1.6-1.
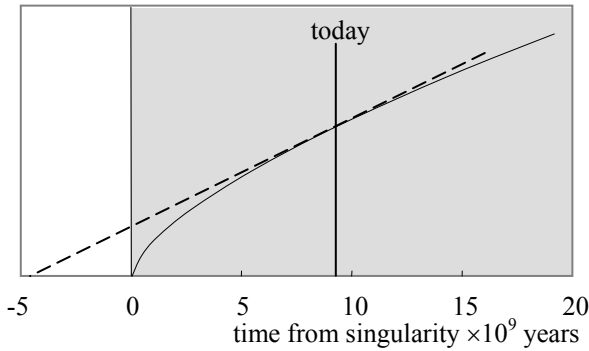

Figure A1.6-1. Accumulation of nuclear decay products at today's decay rate (dashed line), and at a rate proportional to the velocity of light in DU space (solid curve).

A1.7 Microwave Background radiation in DU space

The bolometric energy density of cosmic microwave background radiation, $4.2 \cdot 10^{-14}$ [J/m^3], corresponds, with a high accuracy, to the energy density *within a closed blackbody source* at 2.725 °K. (Obs. As indicated by the Stefan-Boltzmann constant, the energy density *within a blackbody source* is higher than the integrated energy density of the flux radiated by the source by a factor of four.)

$$E_{bol(T=2.725 \ °K)} = E_\nu d\nu = \int_0^\infty \frac{8\pi h}{c^3} \nu_0^3 \frac{(\nu/\nu_0)^3}{(e^{\nu/\nu_0} - 1)} d\nu = 4.2 \cdot 10^{-14} \quad \left[\frac{J}{m^3}\right] \tag{A1.7:1}$$

where

$$\nu_0 \equiv \frac{kT}{h} = \frac{c}{\lambda_0} \quad [Hz] \tag{A1.7:2}$$

from which $\nu_0 = 5.69 \cdot 10^{10}$ Hz is obtained for $T = 2.725$ °K.

The rest energy calculated for the total mass in space is $E_{rest} = M_\Sigma \cdot c^2 \approx 2 \cdot 10^{70}$ [J] corresponding to energy density $E_{rest}/(2\pi^2 R_4^3) = 4.6 \cdot 10^{-10}$ [J/m^3] in DU space. Assuming that CMB is equal everywhere in space, the share of the CMB energy density of the total energy density in space is about 10^{-4}. The total mass equivalence, and hence the ratio to the rest energy in space is conserved. The wavelength of radiation is redshifted as

$$z = \frac{R_4 - R_{4(e)}}{R_{4(e)}} = \frac{R_4}{R_{4(e)}} - 1 \tag{A1.7:3}$$

where $R_{4(e)}$ is the 4-radius of space at the time of the emission of the CMB. The DU concept does not give a prediction for the value of the 4-radius $R_{4(e)}$ at the emission of the CMB — or exclude the possibility that the CMB were generated continuously by dark matter now at 2.725 °K.

Appendix A2. Blackbody radiation

By denoting

$$\lambda_0 \equiv \frac{hc}{kT} \qquad \Rightarrow \qquad \frac{\lambda_0}{\lambda} = \frac{hc}{\lambda kT} \quad ; \quad \lambda = \frac{hc}{kT} \frac{1}{\lambda_0/\lambda} \quad \text{and} \quad h\frac{c}{\lambda_0} = h\nu_0 = kT \qquad \text{(A2:1)}$$

the energy density of a black body source expressed in terms of a wavelength differential $d\lambda$ is

$$dE_\lambda = E(\lambda)d\lambda = \frac{8\pi hc}{\lambda_0^5} \frac{(\lambda_0/\lambda)^5}{(e^{\lambda_0/\lambda}-1)} d\lambda \qquad \left[\frac{J}{m^3}\right] \qquad \text{(A2:2)}$$

or in terms of a frequency differential $d\nu$

$$dE_\nu = E(\nu)d\nu = \frac{8\pi \nu_0^3 h}{c^3} \frac{(\nu/\nu_0)^3}{(e^{\nu/\nu_0}-1)} \cdot d\nu \qquad \left[\frac{J}{m^3}\right] \qquad \text{(A2:3)}$$

The energy flux in terms of a wavelength differential $d\lambda$ or a frequency differential $d\nu$ from a black body source is obtained by multiplying the energy densities in (A2:2) and (A2:3) by the Stefan-Boltzmann factor $c/4$, and further divided by 4π for flux per steradian

$$dF_\lambda = \frac{c}{4\cdot 4\pi} dE_\lambda = \frac{hc^2}{2\lambda_0^5} \frac{(\lambda_0/\lambda)^5}{(e^{\lambda_0/\lambda}-1)} d\lambda = F(\lambda)d\lambda \qquad \left[\frac{W}{m^2 sr}\right]$$

$$F(\lambda) = \frac{hc^2}{2\lambda_0^5} \frac{(\lambda_0/\lambda)^5}{(e^{\lambda_0/\lambda}-1)} = \frac{F_0}{\lambda_0} \frac{(\lambda_0/\lambda)^5}{(e^{\lambda_0/\lambda}-1)} \qquad \left[\frac{W}{m^2 sr}\middle/ m\right] \qquad \text{(A2:4)}$$

for equation (A2:2), and

$$dF_\nu = \frac{c}{4\cdot 4\pi} dE_\nu = \frac{\nu_0^3 h}{2c^2} \frac{(\nu/\nu_0)^3}{(e^{\nu/\nu_0}-1)} d\nu = F(\nu)d\nu \qquad \left[\frac{W}{m^2 sr}\right]$$

$$F(\nu) = \frac{\nu_0^3 h}{2c^2} \frac{(\nu/\nu_0)^3}{(e^{\nu/\nu_0}-1)} = \frac{F_0}{\nu_0} \frac{(\nu/\nu_0)^3}{(e^{\nu/\nu_0}-1)} \qquad \left[\frac{W}{m^2 sr\ Hz}\right] \qquad \text{(A2:5)}$$

for equation (A2:3). Factor F_0 in equations (A2:4) and (A2:5) is

$$F_0 = \frac{(kT)^4}{2c^2 h^3} = \frac{h}{2} \frac{c^2}{\lambda_0^4} = \frac{h}{2} \frac{\nu_0^4}{c^2} \qquad \left[\frac{W}{m^2 sr}\right] \qquad \text{(A2:6)}$$

The total energy flux from a black body source is obtained by integrating (A2:5) or (A2:6) for all wavelengths or frequencies. Substitution of $x = \lambda_0/\lambda$ in (A2:5) or $x = \nu/\nu_0$ in (A2:6) gives

$$F_{bol} = F_0 \int_0^\infty \frac{x^3}{(e^x-1)} dx = \frac{\pi^4}{15} F_0 = \frac{\pi^4}{15} \frac{k^4 T^4}{2c^2 h^3} = \frac{\sigma}{4\pi} T^4 \qquad \left[\frac{W}{m^2 sr}\right] \qquad \text{(A2:7)}$$

where the numerical factor $\pi^4/15$ comes from the definite integral, T is the temperature of the black body source, and σ is the Stefan-Boltzmann constant $\sigma = 5.6693\cdot 10^{-8}$ [Wm^{-2}°K^{-4}].

The energy flux emitted in the wavelength or frequency range of a narrowband filter with relative width $W = W_\lambda = \Delta\lambda/\lambda = W_\nu = \Delta\nu/\nu$ is obtained from equations (A2:4) and (A2:5), respectively,

$$F_{W(\lambda)} = F_0 \frac{(\lambda_0/\lambda)^4}{(e^{\lambda_0/\lambda} - 1)} \frac{d\lambda}{\lambda} = F_0 \frac{(\lambda_0/\lambda)^4}{(e^{\lambda_0/\lambda} - 1)} W \left[\frac{W}{m^2 sr}\right] \tag{A2:8}$$

$$F_{W(\nu)} = F_0 \frac{(\nu/\nu_0)^4}{(e^{\nu/\nu_0} - 1)} \frac{d\nu}{\nu} = F_0 \frac{(\nu/\nu_0)^4}{(e^{\nu/\nu_0} - 1)} W \left[\frac{W}{m^2 sr}\right] \tag{A2:9}$$

or by relating the narrow band power density to the total bolometric flux density by expressing F_0 in terms of F_{bol} (A2:7) as

$$F_{W(\nu,\lambda)} = \frac{15}{\pi^4} \frac{(\nu/\nu_0)^4}{(e^{\nu/\nu_0} - 1)} W \cdot F_{bol} = \frac{15}{\pi^4} \frac{(\lambda_0/\lambda)^4}{(e^{\lambda_0/\lambda} - 1)} W \cdot F_{bol} \left[\frac{W}{m^2 sr}\right] \tag{A2:10}$$

The distribution function $D = x^4/(e^x - 1)$ obtains its maximum value when $x = 3.9207$

$$D_{max} = \left(\frac{x^4}{e^x - 1}\right)_{max} = D_{(x=3.9207)} = 4.780 \tag{A2:11}$$

At a fixed relative bandwidth W the maximum flux occurs when the nominal frequency or wavelength of the filter $f_W = c/\lambda_W$ is $f_W/f_0 = \lambda_0/\lambda_W = 3.9207$

$$F_{W(\nu,\lambda)} = \frac{15}{\pi^4} \cdot D_{max} \cdot W \cdot F_{bol} \left[\frac{W}{m^2 sr}\right] \tag{A2:12}$$

which relates the energy flux through an ideal narrow band filter matched to the bolometric energy flux of the radiation. The nominal frequency of the filter is matched to the maximum power throughput of blackbody radiation by setting $f_W = 3.9207 \cdot f_0$.

When related to the frequency of the maximum power density per frequency f_T [W/Hz/m^2], and at the wavelength of the maximum power density per wavelength λ_T [W/m/m^2], the nominal frequency and wavelength for the maximum power density of blackbody radiation, f_W and λ_W, are, Fig A2-1

$$f_W = \frac{3.9207}{2.8214} = 1.39 \cdot f_{(W/Hz/m^2)} \tag{A2:13}$$

$$\lambda_W = \frac{\lambda_{0(W/m/m^2)}}{3.9207} = \frac{4.9651}{3.9207} = 1.27 \cdot \lambda_{T(W/Hz/m^2)}$$

In terms of the energy per a wavelength or a cycle of electromagnetic radiation equations (A2:8) and (A2:9) can be written in form [see equation (A1.2:2)

$$E_{\nu(W)} = \frac{F_{W(\nu,\lambda)}}{\nu} = \frac{\nu_0^2}{2c^2} \frac{(\nu/\nu_0)^2}{(e^{\nu/\nu_0} - 1)} W \cdot h\nu = I_{(\nu)} \cdot h\nu = I_{(\lambda)} \cdot \frac{h_0}{\lambda} c^2 \tag{A2:14}$$
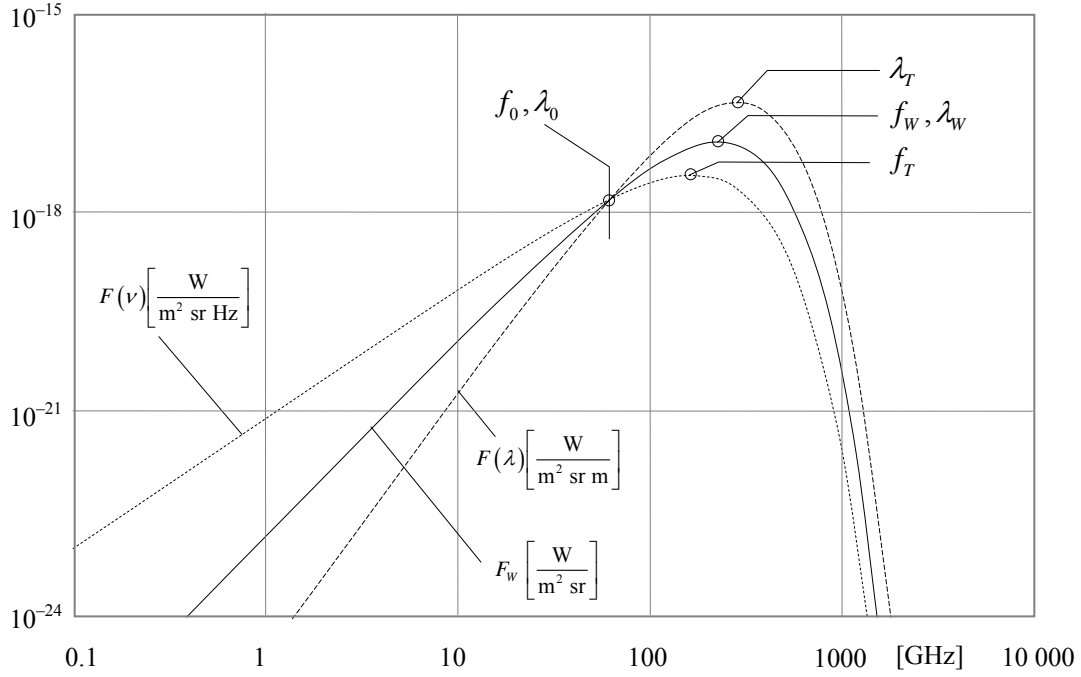
where the intensity factor $I = I_\nu = I_\lambda$ is

Figure B1-1. The energy flux density of black body radiation (CMB) in terms of $F(\lambda)$ [Wm^{-2}sr^{-1}m^{-1}] (A2:4), F_W [Wm^{-2}sr^{-1}] (A2:10), and $F(\nu)$ [Wm^{-2}sr^{-1}Hz^{-1}] (A2:5) in the frequency range from 100 MHz to 10 THz. The wavelength of the observed maximum power density in terms of $F(\nu)$ [Wm^{-2}sr^{-1}Hz^{-1}] is 1.87 mm. In terms of $F(\lambda)$ [Wm^{-2}sr^{-1}m^{-1}] the maximum occurs at wavelength 1.06 mm. The integrated total energy is equal for each flux density functions. Curve F_W [Wm^{-2}sr^{-1}] shows the shape of the flux density function observed in narrow band filters with $W = \Delta\lambda/\lambda = \Delta\nu/\nu$.

$$I_{(\nu)} = I_{(\lambda)} = \frac{\nu_0^2 W}{2c^2} \frac{(\nu/\nu_0)^2}{(e^{\nu/\nu_0} - 1)} = \frac{W}{2\lambda_0^2} \frac{(\lambda_0/\lambda)^2}{(e^{\lambda_0/\lambda} - 1)} \tag{A2:15}$$

Equation (A2:14) shows the energy of a cycle of radiation at wavelength λ receivable with a narrowband filter with relative width $W = \Delta\lambda/\lambda = \Delta\nu/\nu$. The blackbody source is characterized by $\lambda_0 = hc/kT$. In terms of mass equivalence, and by observing the different velocities c and c_0 related to the DU concept, equation (A2:14) is written

$$E_{\nu(W)} = \frac{h_0}{\lambda} c_0 c = \frac{W h_0}{2\lambda_0^3} \frac{(\lambda_0/\lambda)^3}{(e^{\lambda_0/\lambda} - 1)} c_0 c = m_\lambda c_0 c \tag{A2:16}$$

where the mass equivalence of wavelength λ is

$$m_\lambda = \frac{W h_0}{2\lambda_0^3} \frac{(\lambda_0/\lambda)^3}{(e^{\lambda_0/\lambda} - 1)} \tag{A2:17}$$

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